Physics 102 Homework #4

first draft due Wednesday, February 15th
final draft due Sunday, February 19th

1a. If a machine makes a sound that has an intensity of $I = 6 \times 10^{-7}\mathrm{W/m^2}$ where you are (2 meters from the machine), how many decibels is it?

The formula for decibels:

$$\beta = 10(\log_{10} I + 12)$$
$$= 10(\log_{10}(6 \times 10^{-7}) + 12)$$
$$= 10(-6.22 + 12)$$
$$\mathbf{= 57.8dB.}$$

Note: the $\log_{10}$ of a number in scientific notation will be close to its exponent: thus $\log_{10} 6 \times 10^{-7} = -6.2 \approx -7$. Some people treated this logarithm as a very small number, which it is not. Logarithms are weird that way.

1b. If I add a second machine at the same place, how many decibels will I hear then?

The intensity would double:
$\beta = 10(\log_{10} 2I + 12) = 10(-5.92 + 12) =$ **60.8dB.**

Or using the book's explanation, the decibel level of 2 such machines is
$\beta_2 = \beta_1 + 10\log_{10} 2 = 57.8\mathrm{dB} + 3.0\mathrm{dB} =$ **60.8dB.**

1c. If I turn the second machine off, and step backwards until I am 4 meters away from the machine, how many decibels do I hear?

According to the book,

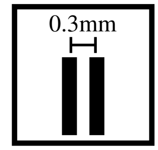
$$\beta(r') = \beta(r) - 20\log_{10}\frac{r'}{r}.$$

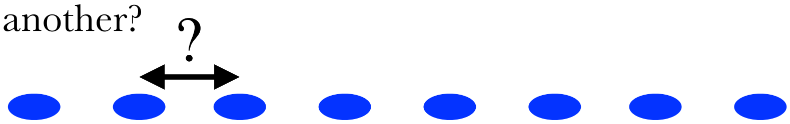
In this case, the original position is $r = 2\mathrm{m}$ and the new position is $r' = 4\mathrm{m}$, so the decibel value at r' is

$\beta(4\mathrm{m}) = \beta(2\mathrm{m}) - 20\log_{10}\frac{4\mathrm{m}}{2\mathrm{m}} = 57.8\mathrm{dB} - 20\log_{10} 2 = 57.8 - 6.0 =$ **51.8dB.**

Or…intensity depends on the distance *squared* from the source, so if you move twice as far away, the intensity drops by 2^2 = one-fourth of the original. Then
$\beta(4\mathrm{m}) = (10\mathrm{dB})(\log_{10}\frac{6 \times 10^{-7}}{4} + 12) = 51.8\mathrm{dB}.$

2a. Blue laser light (400nm) shines through two narrow vertical slits that are 0.3mm apart, onto a screen that is 5 meters away. How far apart are the dots from one another?

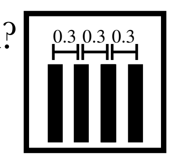




$$\Delta y = \frac{\lambda L}{d} = \frac{(400 \times 10^{-9})(5)}{0.3 \times 10^{-3}} = 6.67 \times 10^{-3}\text{m} = \mathbf{6.67mm}.$$

If y'all can tell me why some of y'all are writing "4.6" for L instead of "5", I'd be most obliged. Can't figure it out…

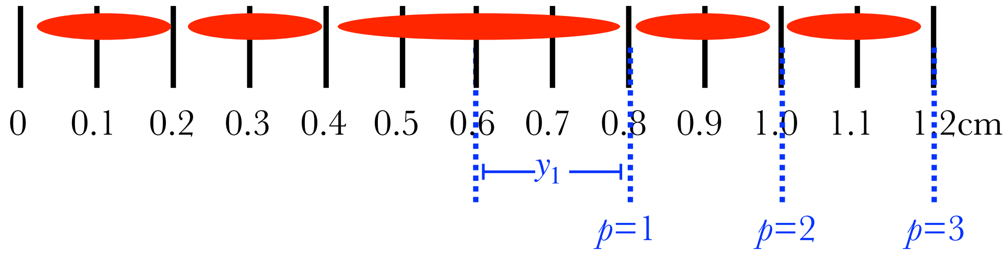
2b. If we add a few more slits as shown, how far apart would the dots be then?



The dots would be at exactly the same positions, only smaller. Thus $\Delta y =$ **6.67mm.**

Some people tried to calculate d by taking the total width of the lines (0.9mm) divided by the number of lines (4), to get $d = 0.9/4 = 0.225$mm instead of 0.3mm. This procedure works fine if there are a lot of lines, but if we know the actual distance between each pair of lines, it's better to just use that.

3. Red laser light (633nm) shines through a single slit onto a screen that is 3 meters away, and creates the pattern shown. How wide is the slit, in millimeters?



This is *diffraction* from a single slit, so we label the *dark* spots.
The distance from the center (0.6cm) to the first dark spot (0.8cm) is $y_{p=1} = 0.2\text{cm} = 0.2 \times 10^{-2}\text{m}$. Since $L = 3\text{m}$ and $\lambda = 633\text{nm}$, we have

$$y_p = \frac{p\lambda L}{a} \implies a = \frac{p\lambda L}{y_p} = \frac{1(633 \times 10^{-9}\text{m})(3\text{m})}{0.2 \times 10^{-2}\text{m}} = 9.49 \times 10^{-4}\text{m} = \mathbf{0.949mm.}$$

I've been seeing people writing "6.33×10^{-7}m" for the laser light, and while this is true, there's no reason we can't just keep it as "633×10^{-9}m" and one very good reason to do so: one less potential error! Don't worry about standardizing scientific notation if you're just going to plug it into your calculator.

Remember $1\text{cm} = 10^{-2}\text{m}$: that's a "2" not a "3". ($1\text{mm}=10^{-3}\text{m}$).

4. A visible-light telescope (λ=500nm) can barely distinguish between a pair of binary stars which are 5×10^{-6} radians apart. What is the diameter of the telescope's opening?

The minimum angular separation between two objects (in **radians**) that can be determined is $\theta = 1.22\dfrac{\lambda}{D}$.

Thus $\theta = 1.22\dfrac{\lambda}{D} \implies D = 1.22\dfrac{\lambda}{\theta} = 1.22\dfrac{500 \times 10^{-9}}{5 \times 10^{-6}} = \mathbf{0.12m}.$