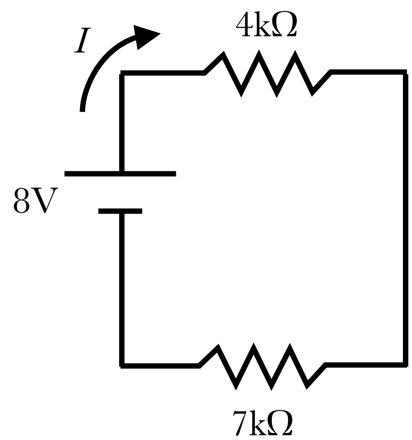
Physics 102 Homework #9

first draft due Wednesday, April 12th
final draft due Sunday, April 16th

1. Find the current in this circuit.



Starting from the negative end of the battery, we can build a loop equation:

$$+8 - 4\text{k}I - 7\text{k}I = 0$$

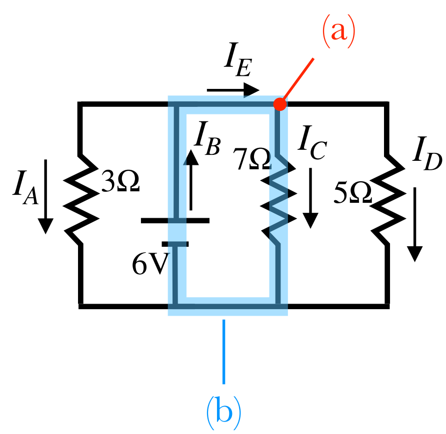
We solve that for I by first moving both I terms to the right side:

$8 = 4\text{k}I + 7\text{k}I = 11\text{k}I$

$\implies I = \dfrac{8}{11\text{k}} = 7.2 \times 10^{-4}\Omega = \mathbf{0.72mA}.$

(Handy tip: when it comes to metric prefixes, 1/k = m.)

2. Consider this circuit.



a) Write a junction rule equation for the junction marked (a).

$$I_E = I_C + I_D$$

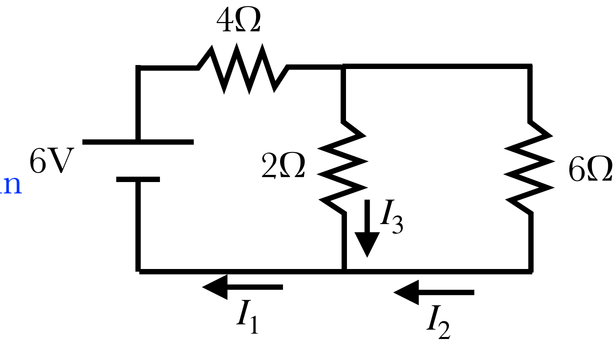
b) Write a loop rule equation for the loop marked (b).

$$6 - 7I_C = 0$$

c) Find the current I_A.

$$6 - 3I_A = 0 \implies I_A = 2\text{A}$$

3a. Write a junction rule equation for this circuit.



The junction on the bottom has currents I_2 and I_3 in and I_1 out. Thus

$$I_2 + I_3 = I_1.$$

3b. Write two loop rule equations for this circuit.

There are three loop equations, but I only need two:

Left loop: $6 - 4I_1 - 2I_3 = 0$

Big loop: $6 - 4I_1 - 6I_2 = 0$

Right loop: $-2I_3 + 6I_2 = 0$

3c. Use your equations to solve for I_1, I_2, and I_3 in this circuit, and fill in the table.

I'm going to rewrite the other variables in terms of I_2. To do that, I use the right loop equation, solving it for I_3 to get $I_3 = 3I_2$. Then I substitute that into the junction equation to get

$I_1 = I_2 + 3I_2 \implies I_1 = 4I_2$.

Now I can substitute these two results into the big loop equation to get a single equation in terms of I_2, and then solve that:

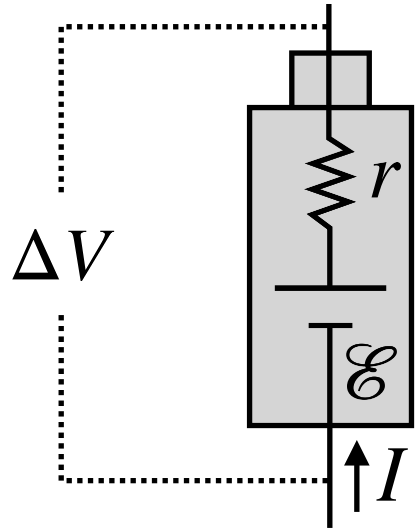
$$6 - 4(4I_2) - 6I_2 = 0 \implies 6 = 22I_2 \implies I_2 = \mathbf{0.27A}.$$

Now I can substitute this into the other two equations I solved, giving me

$$I_3 = 3I_2 = \mathbf{0.81A} \quad \text{and} \quad I_1 = 4I_2 = \mathbf{1.08A}.$$

I_1	**1.08A**
I_2	**0.27A**
I_3	**0.81A**

4. Real batteries have an internal resistance r in addition to its emf $\mathscr{E}$. Suppose we have a real battery with $\mathscr{E} = 9\text{V}$ and an internal resistance $r = 5\Omega$.



a. What is the potential difference ΔV across the ends of this battery, as a function of the current I through it?

$\Delta V = \mathscr{E} - Ir = 9 - 5I$

b. What is the maximum amount of current I_{max} that can be produced by this battery?

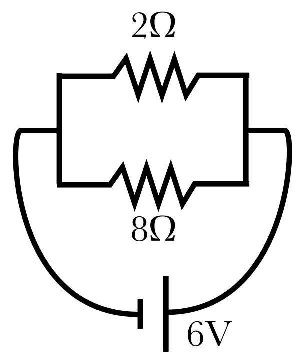
The voltage ΔV across the battery has to be positive for any current to flow. Thus $\Delta V = \mathscr{E} - Ir \geq 0 \implies I \leq \dfrac{\mathscr{E}}{r} = \dfrac{9\text{V}}{5\Omega} = 1.8\text{A}$, which is I_{max}.

c. The power output of the battery is $P = I\Delta V$. The maximum power output by the battery when $I = \frac{1}{2}I_{max}$. Find the maximum power output of the battery.

The power output is $P = I\Delta V = I(\mathscr{E} - Ir) = \mathscr{E}I - I^2 r$. If $I = \dfrac{1}{2}I_{max} = \dfrac{\mathscr{E}}{2r}$, then $P = \mathscr{E}\dfrac{\mathscr{E}}{2r} - \left(\dfrac{\mathscr{E}}{2r}\right)^2 r = \dfrac{\mathscr{E}^2}{2r} - \dfrac{\mathscr{E}^2}{4r} = \dfrac{\mathscr{E}^2}{4r}$. Or in our case,

$P = \dfrac{9^2}{4(5)} = 4.05\text{W}$.

5a. What is the equivalent resistance of these two resistors?



The equivalent conductance (reciprocal resistance) of this pair of parallel resistors is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2}\mho + \frac{1}{8}\mho = \frac{5}{8}\mho$$

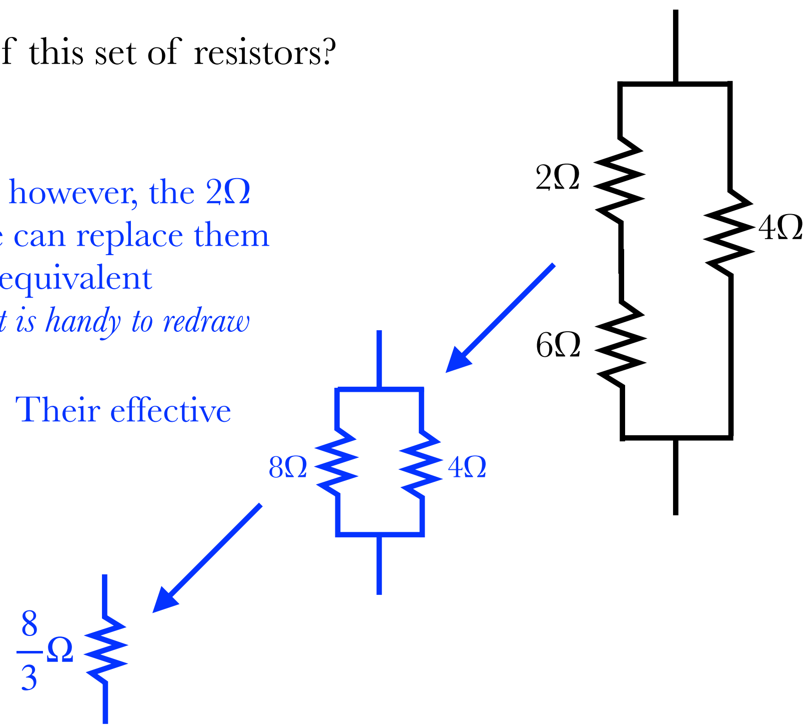
The equivalent resistance is found by flipping both sides upside-down:

$$R_{eq} = \frac{8}{5}\Omega = \mathbf{1.6\Omega}.$$

5b. Use the equivalent resistance to find the current through the 6V battery.

The potential drop across the pair of resistors is ΔV=6V, and so the current that runs through both of them combined is $I = \frac{\Delta V}{R_{eq}} = \frac{6\text{V}}{1.6\Omega}$=**3.75A**.

6. What is the equivalent resistance of this set of resistors? Use resistance reduction.



There are no resistors in parallel (yet); however, the 2Ω and 6Ω resistor are in series. Thus we can replace them with a single resistor having the same equivalent resistance $R_{eq} = 2\Omega + 6\Omega = 8\Omega$. *I find it is handy to redraw the circuit with the replacement added.*

Now we have two resistors in parallel. Their effective conductance is

$$\frac{1}{R_{eq}} = \frac{1}{8\Omega} + \frac{1}{4\Omega} = \frac{3}{8}\mho$$

and so

$$R_{eq} = \frac{8}{3}\Omega = \mathbf{2.67\Omega.}$$