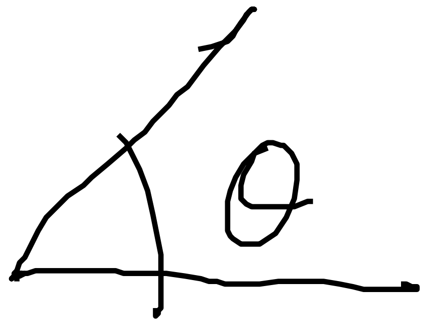
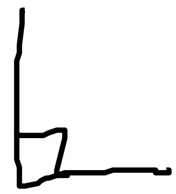
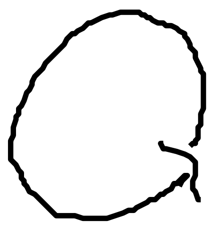
Angular Units

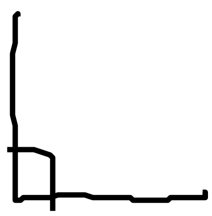


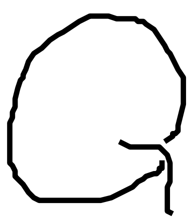
- degrees

 90°

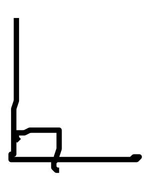
 360°

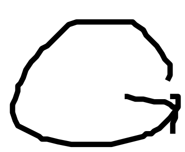
- revolutions or turns

 $\frac{1}{4}$ rev

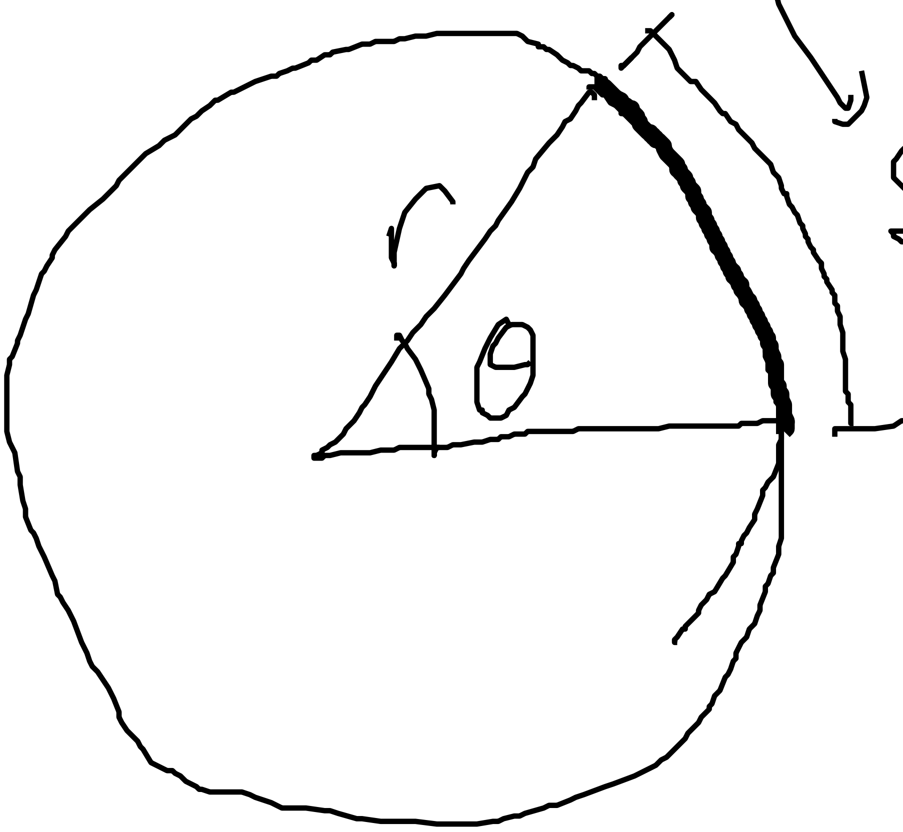
 1 rev

- radians (standard)

 $\frac{\pi}{2}$ rad

 2π rad

Arc Length



$s = r\theta$ only if θ is in radians

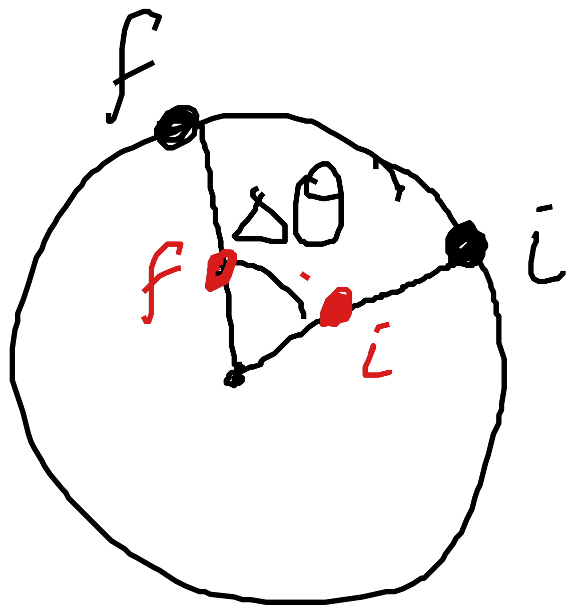
$\theta = \pi$

$$s = \frac{1}{2}\left(\overbrace{2\pi r}^{\text{circumference}}\right)$$

$$= \pi r$$

Angular Displacement

$\Delta\theta$



units: radians

degrees

revs.

On a rigid disk, every point goes through the same angular displacement.

Angular Velocity

$$\omega = \frac{d\theta}{dt}$$

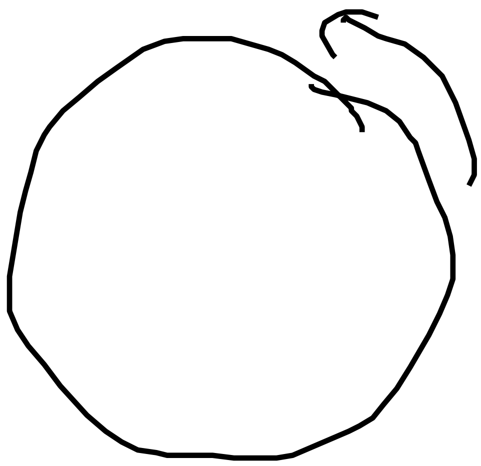
lowercase omega

Units: $\frac{\text{radians}}{s}$

deg/s, rev/s

rpm

When measured in rev/s often referred to as "frequency" f



$$\omega = \frac{5\,\text{rev}}{s}$$

it spins 5 times every second.

$$f = 5\,\frac{\text{rev}}{s} = 5/s$$

$$= 5\,\text{Hz}$$ Hertz

Period $\boxed{T = \frac{1}{f}}$

e.g. $f = 5\,\text{Hz}$, $T = 0.2\,s/\text{rev}$

how long does it take to go around once

If ω is measured in rad/s & f in rev/s,

$$\boxed{\omega = 2\pi f}$$

Angular Acceleration

alpha $\rightarrow$ $\alpha = \frac{d\omega}{dt}$

Units $\quad rad/s^2$

Correspondence

Linear		Angular
Δx	$\longleftrightarrow$	$\Delta\theta$
v	$\longleftrightarrow$	ω
a	$\longleftrightarrow$	α
$\frac{dv}{dt}$	$\longleftrightarrow$	$\frac{d\omega}{dt}$
$\frac{dx}{dt}$	$\longleftrightarrow$	$\frac{d\theta}{dt}$

Constant Angular Acceleration

$$\Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$\hookrightarrow \Delta \theta = \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

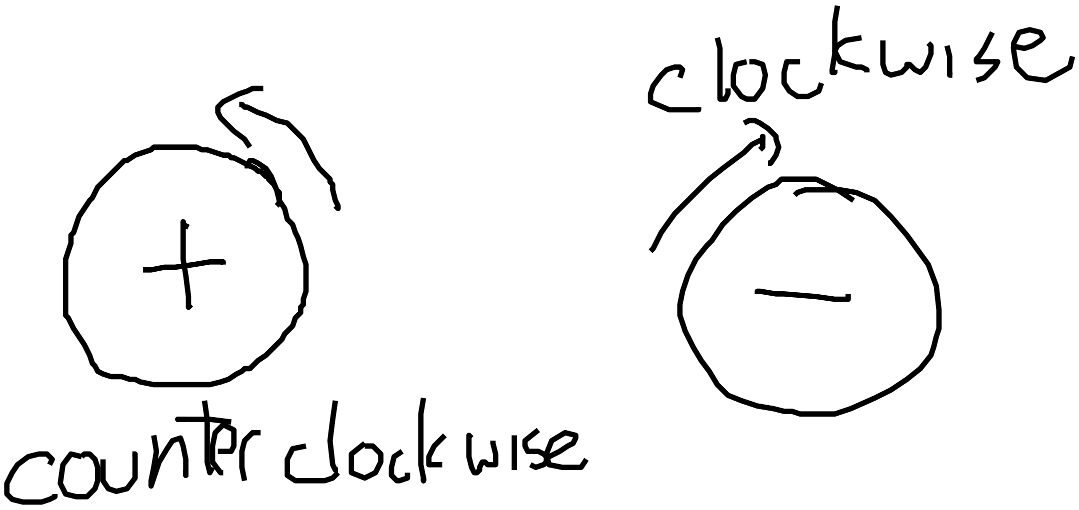
$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\hookrightarrow \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

etc

Signs

in a plane



If ω & α have same sign, speeding up

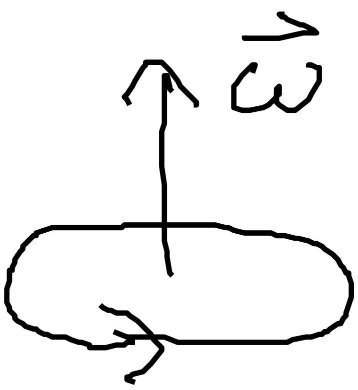
$$\omega_f = \omega_i + \alpha \Delta t$$

If ω & α opposite signs, slowing down

In 3D, rotations can be described with a vector that points along axis of rotation

Right Hand Rule (RHR)

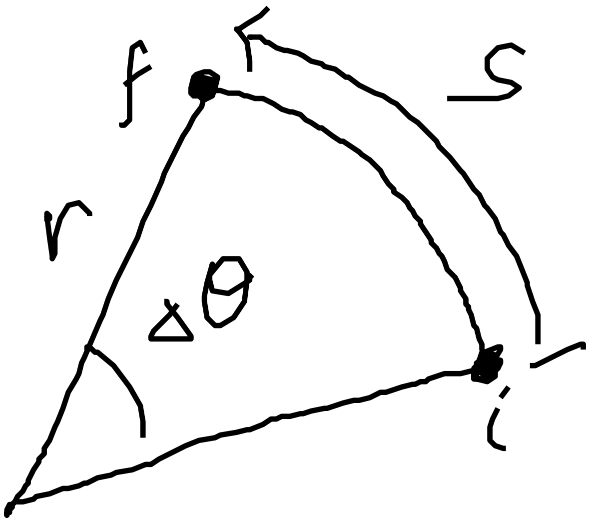
- curl fingers in direction of rotation
- thumb points along the vector



$$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\Delta t$$

$$\omega_i + \vec{\alpha}\Delta t = \vec{\omega}_f$$

(for later)



radius

distance travelled

$$\Delta s = r\,\Delta\theta$$

← radians

$$\frac{\Delta s}{\Delta t} = r\,\frac{\Delta \theta}{\Delta t}$$

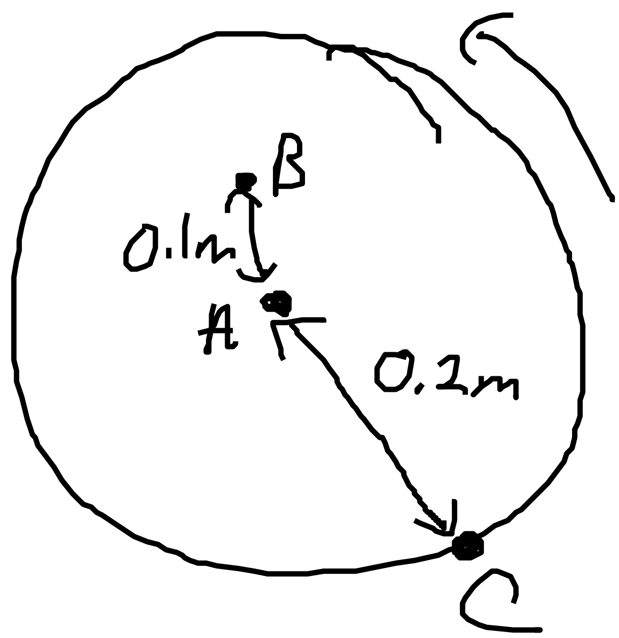
rad/s

$$\boxed{v = r\omega}$$

$$\frac{m}{s} = m\,\frac{\cancel{rad}}{s}$$

← rad can vanish in unit conversions

$$\frac{m}{s} = \frac{m}{s}$$



$\omega = 1\ rad/s$

$1\ rad \approx 57^\circ$

Which point has largest linear speed v?

Every point on a rigid disk moves with the same angular velocity ω,

r: distance from center

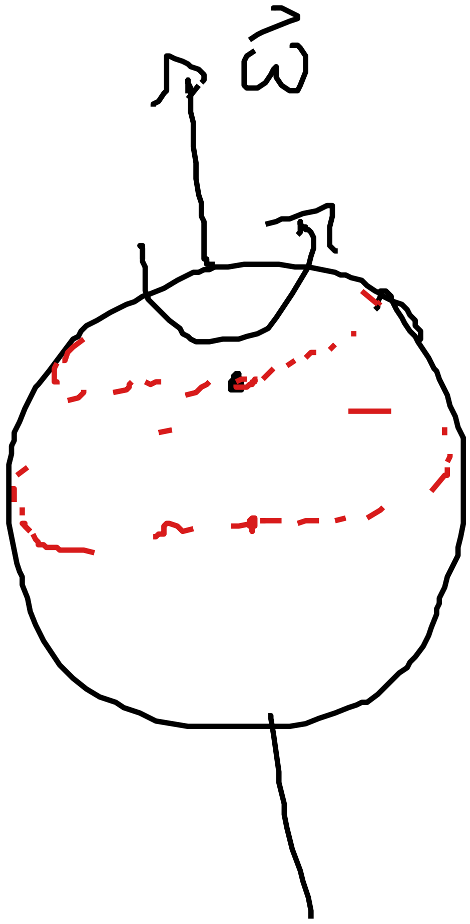
$v = r\omega$

bigger bigger const

$$V_A = (0\,m)(1\ rad/s)$$
$$= 0\ m/s$$

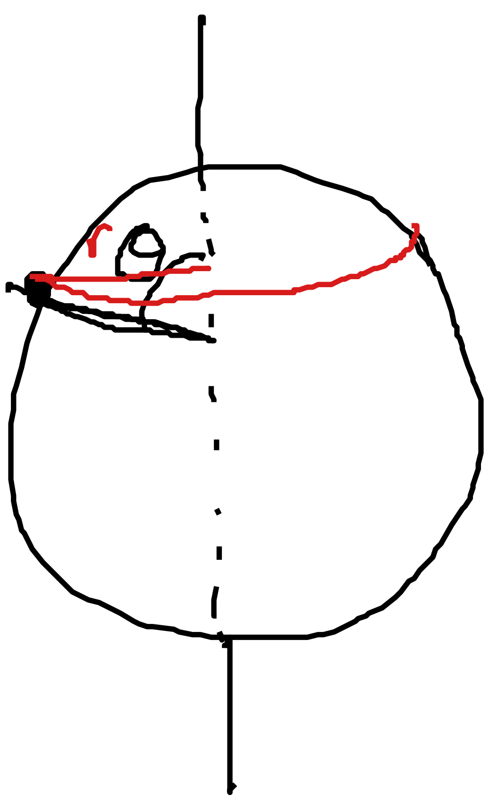
$$V_B = (0.1\,m)(1\ rad/s)$$
$$= 0.1\ m/s$$

$$V_C = (0.2\,m)(2\ rad/s)$$
$$= 0.2\ m/s$$



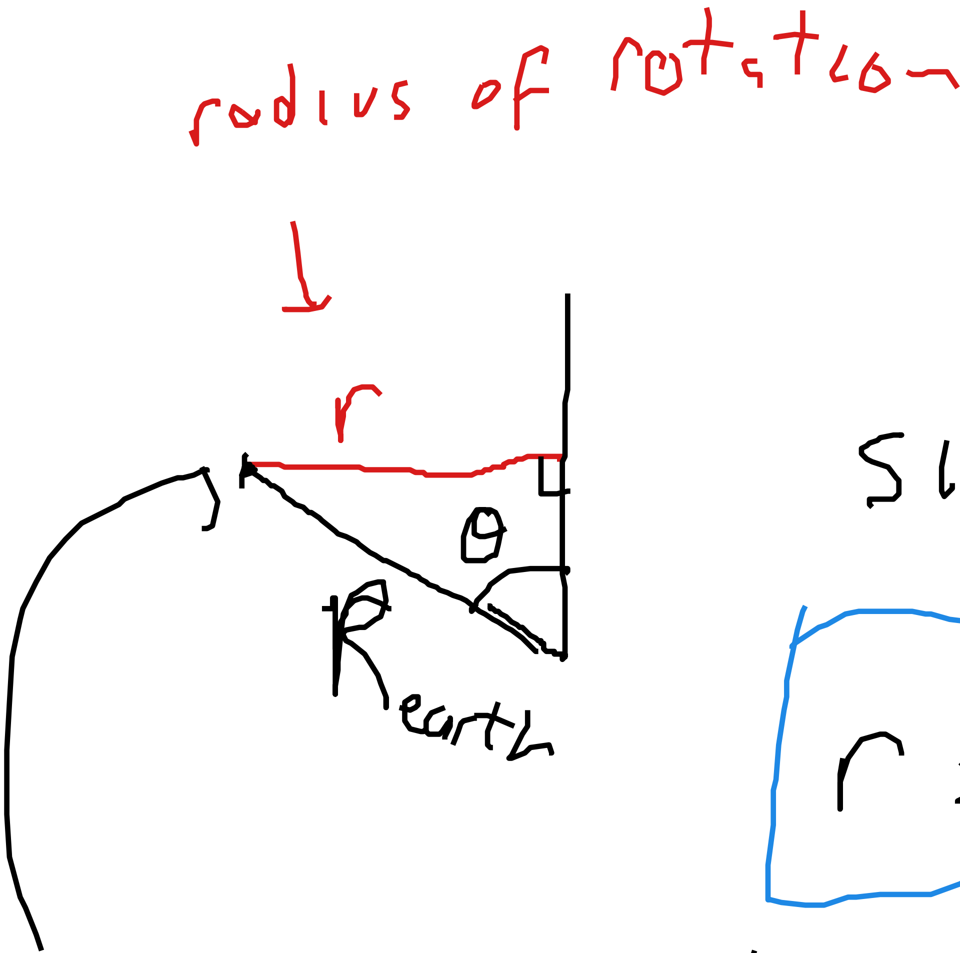
Points on Earth travel around their latitude line.

Equator is the longest latitude line, so points on the equator move the fastest.



θ: angle below the north pole

$= 90^\circ - \text{latitude}$



$$\sin\theta = \frac{r}{R_{earth}}$$

$$\boxed{r = R_{earth}\sin\theta}$$

How fast is this point moving?

$$v = r\omega$$
$$= R_{earth}\,\omega\sin\theta$$

For Toledo, $41.6^\circ N$

$$\theta = 90 - 41.6$$
$$= 48.4^\circ$$

What is ω? rad/s

f rev/s

$$\frac{s}{rev} \rightarrow T = \frac{(24\,hr)(60\,\frac{min}{hr})(60\,\frac{s}{min})}{1\,rev}$$

$$T = 86{,}400\ s/rev$$

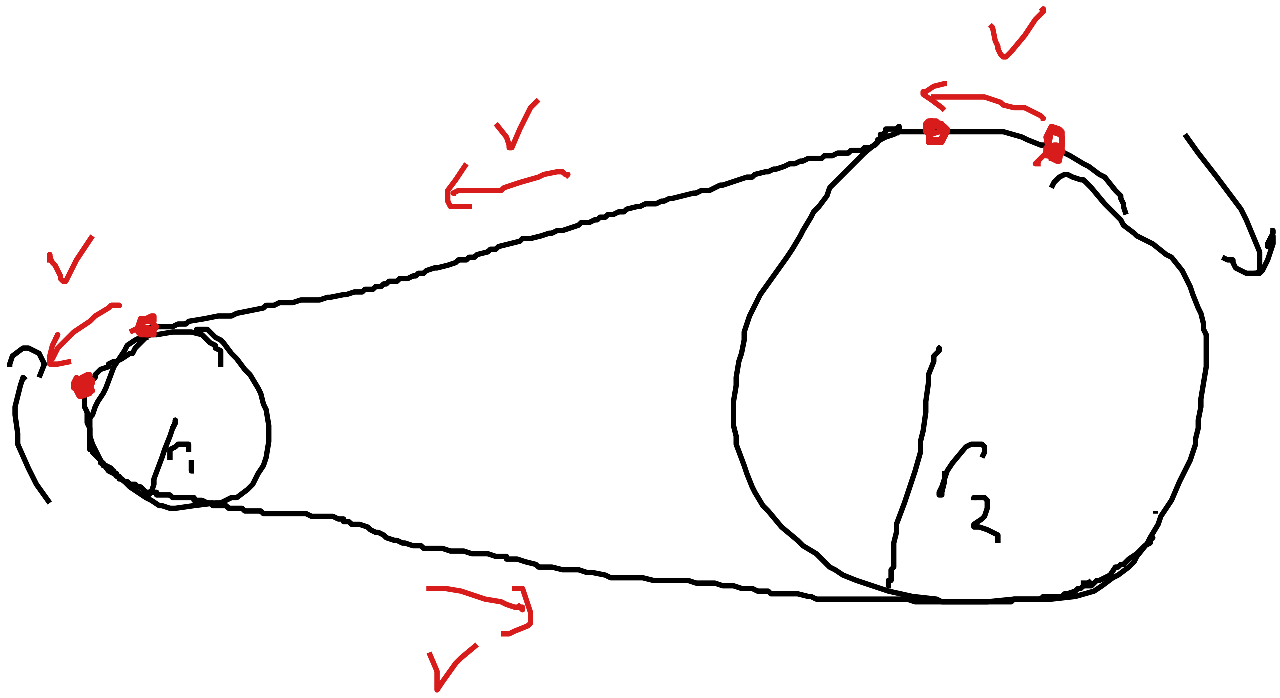
$$f = \frac{1}{86{,}400}\ \frac{rev}{s}$$

$$\omega = \frac{1}{86400}\ \frac{rev}{s} \times \frac{2\pi\ rad}{1\ rev}$$

$$= 7.27 \times 10^{-5}\ rad/s$$

$$v = (6371 \times 10^{3}\,m)(7.27 \times 10^{-5}) \sin 48.4^{\circ}$$

$$= 346\ m/s$$



Which one will have the greater angular velocity ω?

Smaller wheel does.

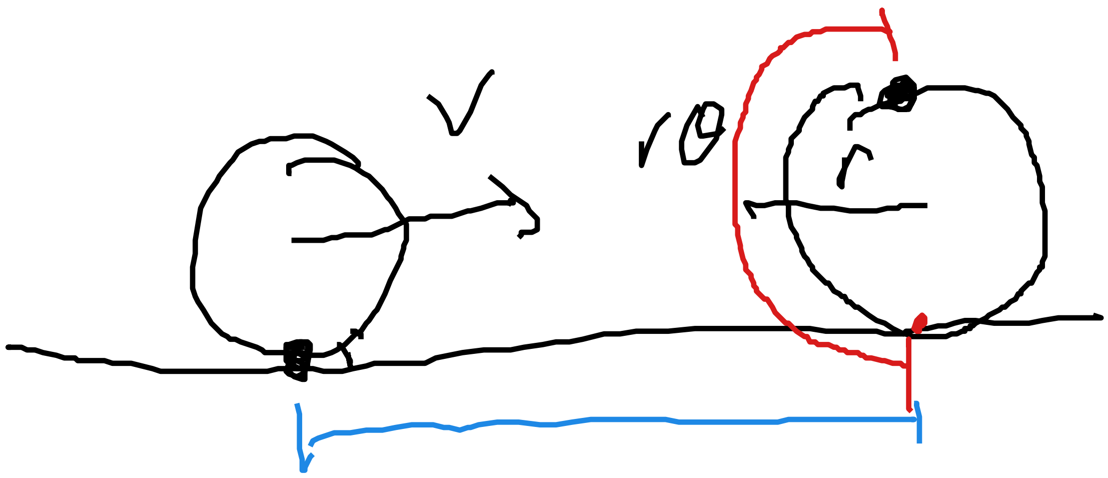
$$v = r\omega$$

$\bar{v}$ is same everywhere along the belt

$$v = r\omega \leftarrow \text{bigger}$$

v: const

r: smaller



Black point has moved a distance $r\theta$ along surface of the wheel

Distance the wheel moves is also $r\theta$ because if wheel didn't sleep, it makes a 1-to-1 correspondence with the road.

Wheel travelled a distance $r\theta$. Let's say it took time t to do it in.

Linear Speed of the wheel (and any car it's attached to) is

$$v = \frac{d}{t} = \frac{r\Delta\theta}{\Delta t}$$

$$v = r\omega$$

If a car is travelling with speed v, its wheels spin with angular velocity $\omega = \frac{v}{r}$

where r = radius of wheels.