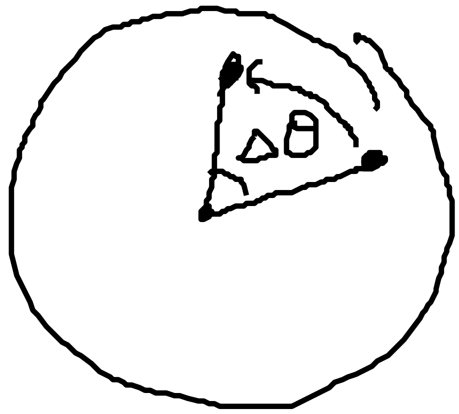


$\Delta\theta$ angular displacement — Δx

ω angular velocity — v

α angular acceleration — a

alpha, not a

Problems with Constant Angular Acceleration

DKDC	
$\Delta\theta$	$\omega_f = \omega_i + \alpha(\Delta t)$
ω_i	$\Delta\theta = \omega_f \Delta t - \frac{1}{2}\alpha(\Delta t)^2$
ω_f	$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$
α	$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$
Δt	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Given 3 of these variables, we can solve for a 4th, leaving one behind. DKDC variable (don't know don't care)

$\Delta\theta$: "turns 180°"

"turns 5 radians"

"turns 50 times"

convert to radians

ω: "spins at 8 rad/s"

"8 times per second" $8 \frac{rev}{s}$

"takes 6s to go around once" $6 \, s/rev$

$\frac{1}{6} \frac{rev}{s}$

$\frac{2\pi}{6} \frac{rad}{s}$

α: rad/s^2

Δt: seconds from beginning to end of motion

A wheel is spinning with an initial angular velocity of $3\,\mathrm{rad/s}$. A brake is then applied, giving it an angular acceleration of $-0.1\,\mathrm{rad/s^2}$. How long does it take for the wheel to go around 5 times?

$\Delta\theta$	5 rev = 10π rad
ω_i	3 rad/s
ω_f	DKDC
α	-0.1 rad/s^2
Δt	NEED

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$10\pi = 3\Delta t + \frac{1}{2}(-0.1)(\Delta t)^2$$

$$0 = \underbrace{-0.05}_{a}(\Delta t)^2 + \underbrace{3}_{b}(\Delta t) \underbrace{-31.4}_{c}$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(-0.05)(-31.4)}}{2(-0.05)}$$

$$= \frac{-3}{-0.1} \pm \frac{1}{-0.1}\sqrt{9 - 6.28}$$

$$= 30 \pm 16.5 \text{ s}$$

$$= \boxed{13.5 \text{ s}} \text{ or } 46.5 \text{ s}$$

wheel stops and reverses until the net # of times around is 5 again

A wheel initially spins so that it takes 6s to go around once. A brake is then applied, and the wheel spins 4 times before it comes to a stop.

What is its angular acceleration?

$6\ s/rev$

$\Delta\theta$ $4\,rev = 8\pi\,rad$

ω_i $6\ s/rev \rightarrow \frac{1}{6}\frac{rev}{s} = \frac{\pi}{3}\frac{rad}{s}$

ω_f 0

α NEED

Δt DKDC

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

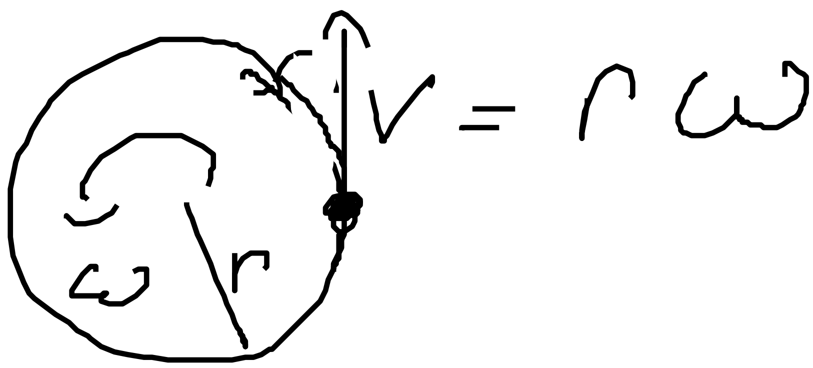
$$0 = \left(\frac{\pi}{3}\right)^2 + 2\alpha(8\pi)$$

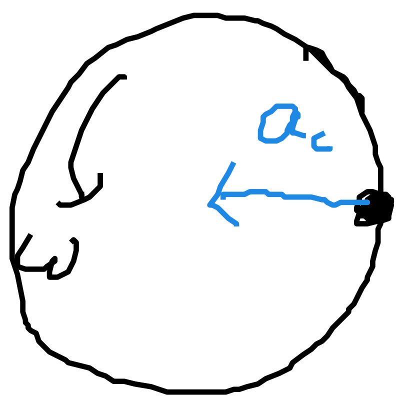
$$-\frac{\pi^{\cancel{2}}}{9} = 16\cancel{\pi}\alpha$$

$$-\frac{\pi}{144} = \alpha$$

$$\alpha = -0.02\ \text{rad}/s^2$$

Angular Acceleration





ω is constant → $\alpha = 0$

what is point's linear acceleration a?

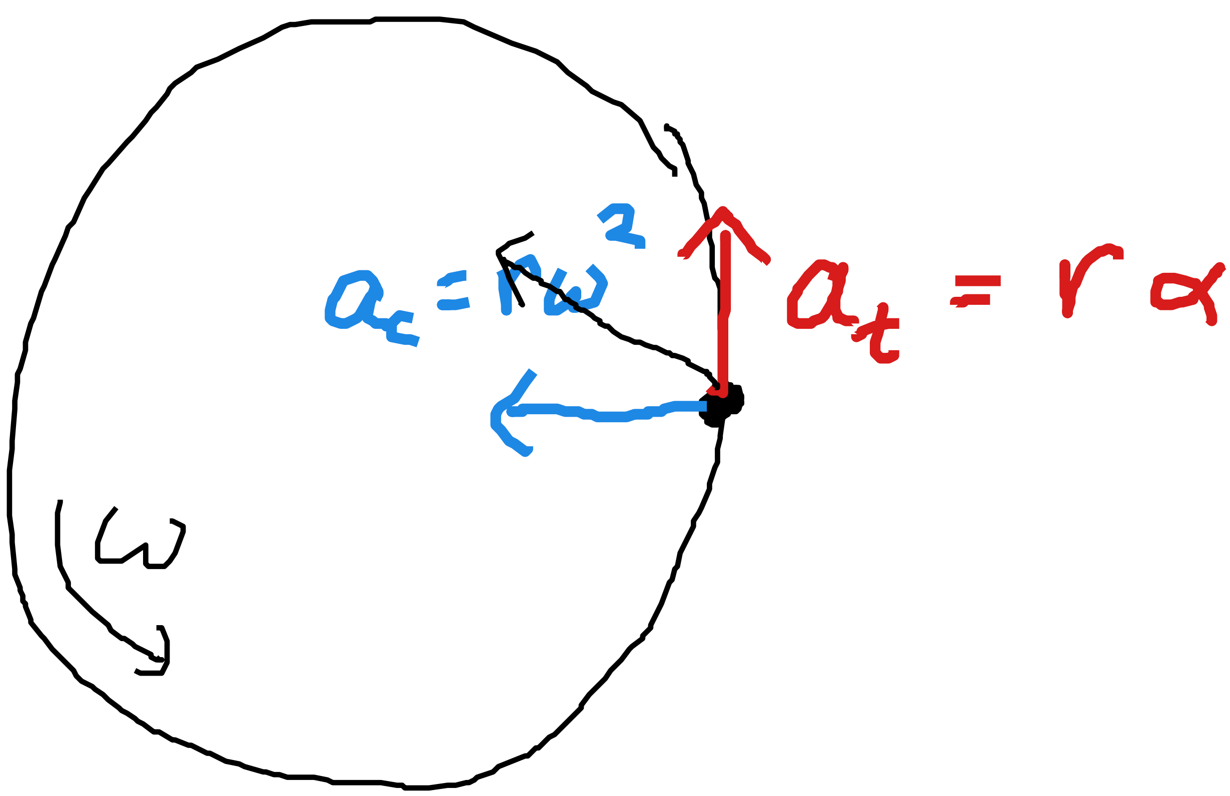
$$a_c = \frac{v^2}{r}$$ centripetal acceleration

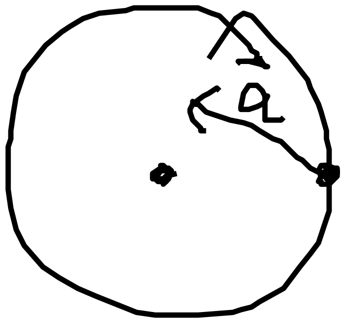
$$a_c = \frac{(r\omega)^3}{r} = \frac{r^{\cancel{3}\,2}\omega^2}{\cancel{r}}$$

$$\boxed{a_c = r\omega^2}$$

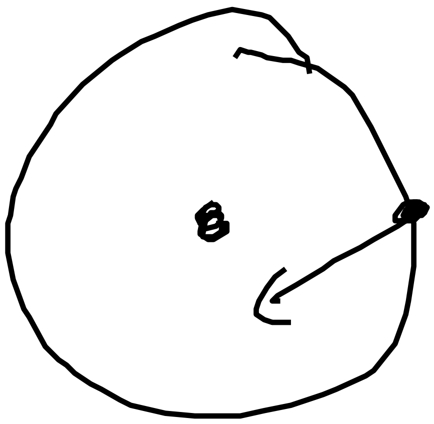
useful because ω is uniform everywhere on rigid spinning object

If $\alpha > 0$

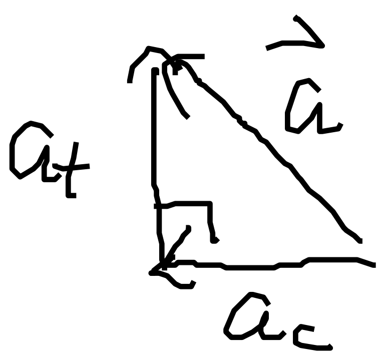




speeding up



slowing down



$$|\vec{a}| = \sqrt{a_c^2 + a_t^2}$$

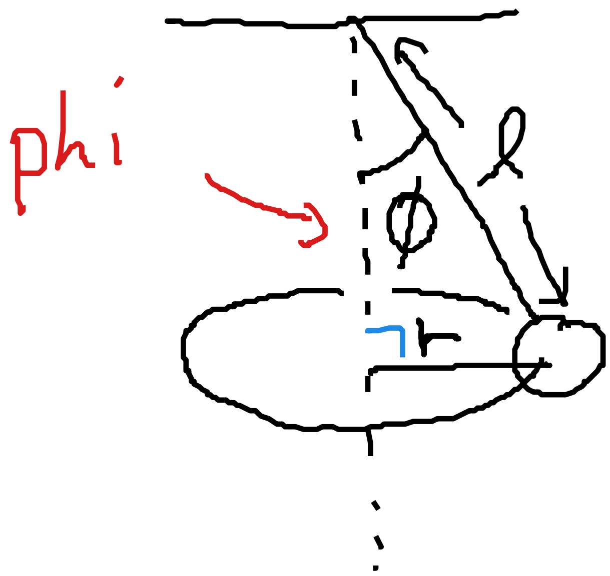
$$= \sqrt{(r\omega^2)^2 + (r\alpha)^2}$$

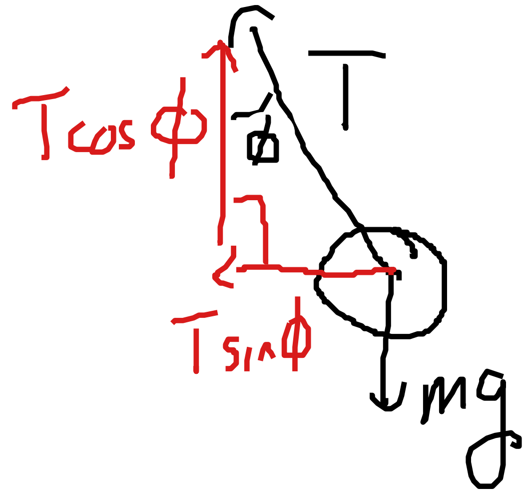
$$= \sqrt{r^2\omega^4 + r^2\alpha^2}$$

$$= \sqrt{r^2}\sqrt{\omega^4 + \alpha^2}$$

$$|\vec{a}| = r\sqrt{\omega^4 + \alpha^2}$$

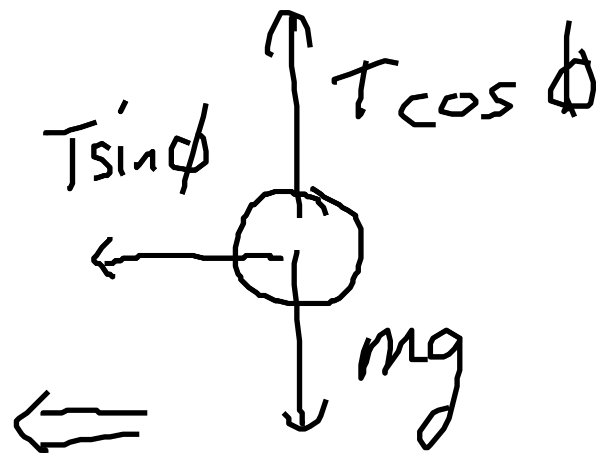
Conical Pendulum





Vertical forces

$$\boxed{mg = T\cos\phi}$$



$a = r\omega^2$

Horizontal forces

$$F_{net} = ma$$

$$\boxed{T\sin\phi = mr\omega^2}$$

$$r = \ell\sin\phi$$

$$T\cancel{\sin\phi} = m(\ell\cancel{\sin\phi})\omega^2$$

$$T = m\ell\omega^2$$

$$\cancel{m}g = T\cos\phi$$

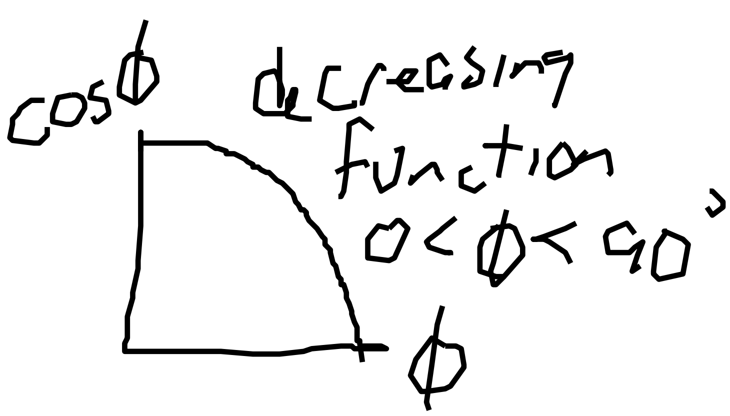
$$= (\cancel{m}\ell\omega^2)\cos\phi$$

$$g = \ell\omega^2\cos\phi$$

$$\omega^2 = \frac{g}{\ell\cos\phi} \qquad \cos\phi = \frac{g}{\ell\omega^2}$$

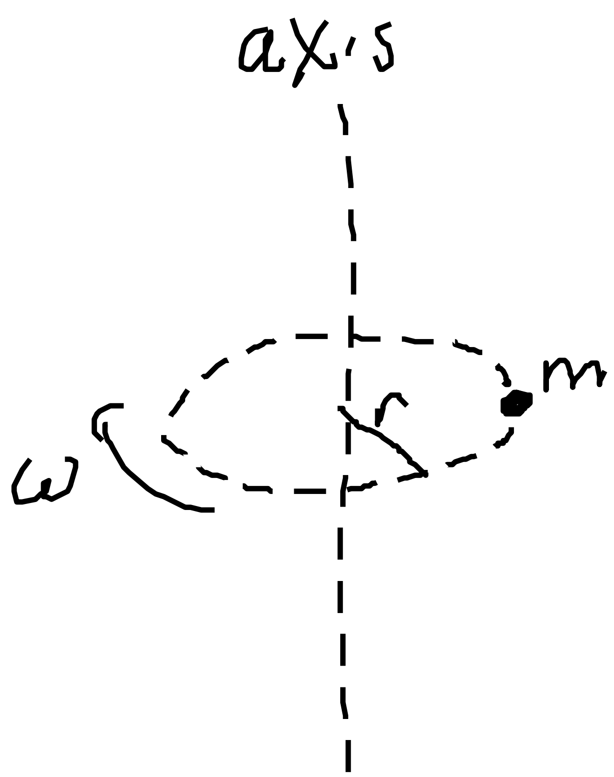
$$\omega = \sqrt{\frac{g}{\ell\cos\phi}}$$

As ℓ bigger, ω gets smaller for same ϕ



As ϕ gets bigger,
$\rightarrow \cos\phi$ gets smaller
$\rightarrow \omega$ gets bigger

Kinetic Energy of Rotation

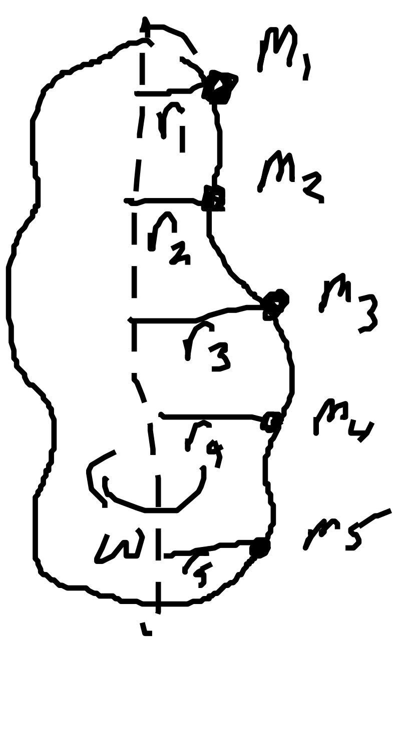


r: distance to axis

$v = r\omega$

$$E_k = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m r^2 \omega^2$$



$$E_k = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \cdots + \frac{1}{2} m_5 r_5^2 \omega^2$$

If all points spin together, ω is same for all of them

$$E_K = \frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2 + \dots + m_5 r_5^2\right)\omega^2$$

$$= \frac{1}{2}\underbrace{\left(\sum_{i=1}^{N} m_i r_i^2\right)}_{I}\omega^2$$

moment of inertia of these points

$$E_K = \frac{1}{2} I \omega^2$$

compare $\frac{1}{2} m v^2$

Moment of inertia is angular analog to mass.
"Rotational inertia"

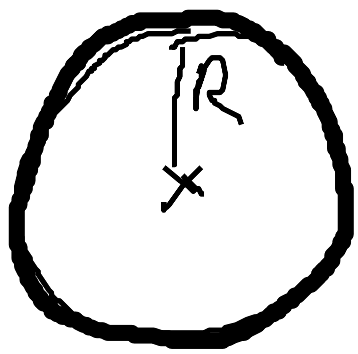
$$I = \sum_{i=1}^{N} m_i r_i^2$$ for a collection of point masses

depends on your choice of axis

for a continuous collection of masses, divide it into a bunch of tiny little masses dm

$$I = \int_{\text{total mass}} r^2 \, dm$$ (we won't use this)

Ring (a circle without its interior) around its central axis with radius R

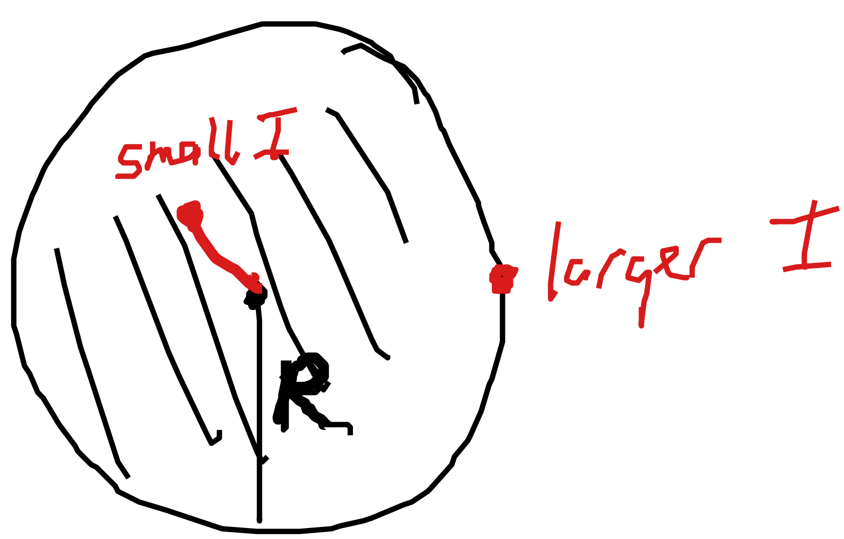


$$I = MR^2$$

M: total mass

Disk (circle with its interior)

$$I = \frac{1}{2} MR^2$$

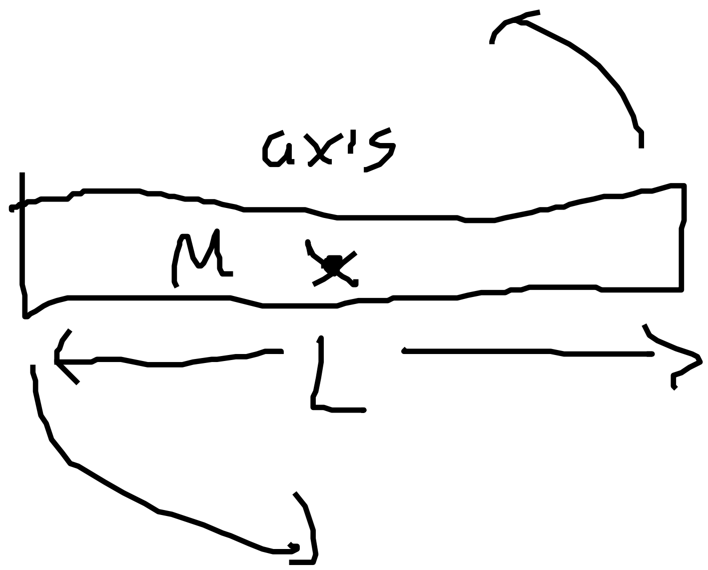


Basically, the more far-flung the mass, the larger the moment of inertia

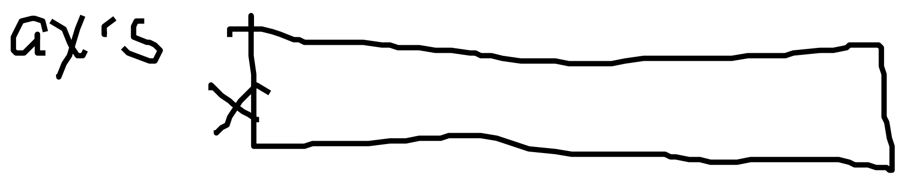
Solid sphere $I = \frac{2}{5} MR^2$

Hollow Sphere $I = \frac{2}{3} MR^2$

Spun around central axis



$$I = \frac{1}{12} ML^2$$



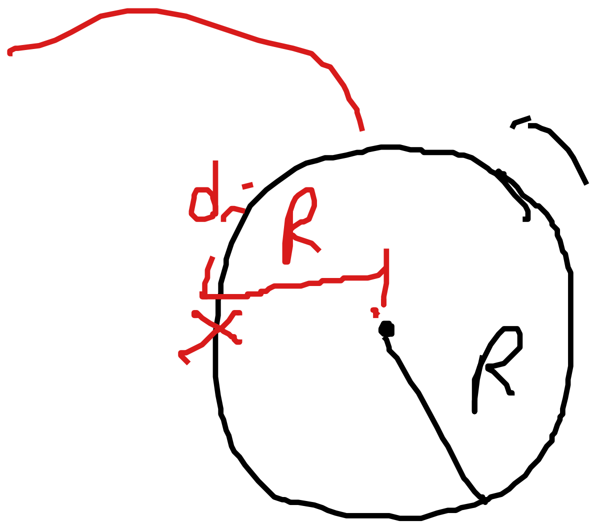
Parallel-Axis Theorem

if you rotate an object around an axis that isn't through the center of mass,

$$I = I_{com} + Md^2$$

(axes must be parallel)

d: distance from C.O.M. to new axis

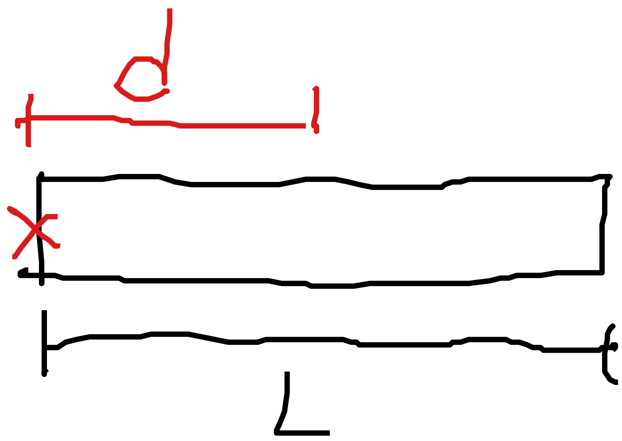


$$I_{com} = MR^2$$

$$I = MR^2 + Md^2$$

$$= MR^2 + MR^2$$

$$= 2MR^2$$



$$I_{com} = \frac{1}{12} ML^2$$

$$I = I_{com} + Md^2$$

$$= \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} ML^2 + \frac{1}{4} ML^2$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) ML^2$$

$$\frac{1}{12} + \frac{3}{12}$$

$$\frac{4}{12} = \frac{1}{3}$$

$$\boxed{I = \frac{1}{3} ML^2}$$

$$E_k = \underbrace{\frac{1}{2} I \omega^2}_{\text{rotational kinetic energy}} + \underbrace{\frac{1}{2} M v_{com}^2}_{\text{linear kinetic energy}}$$