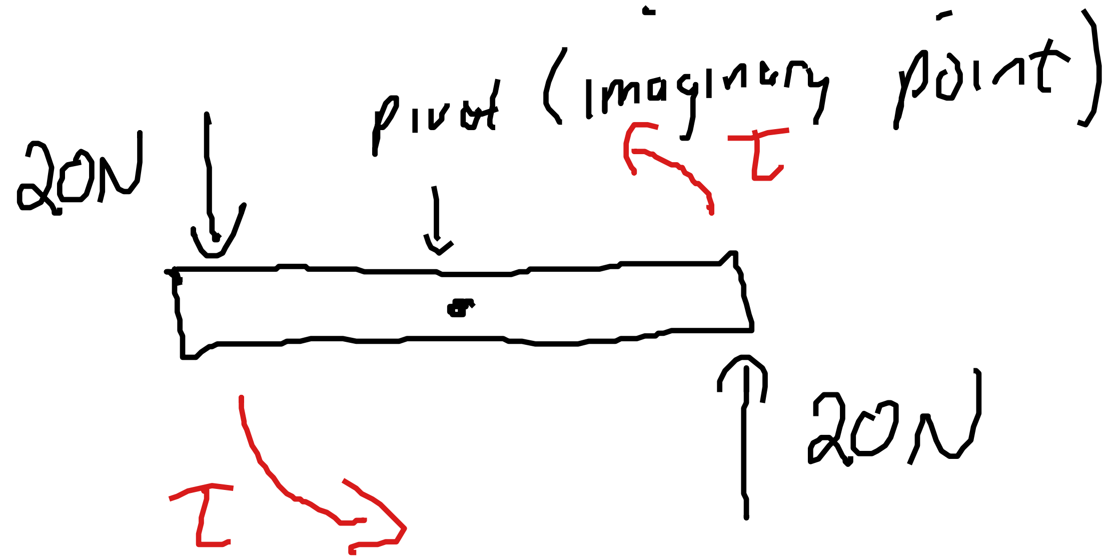
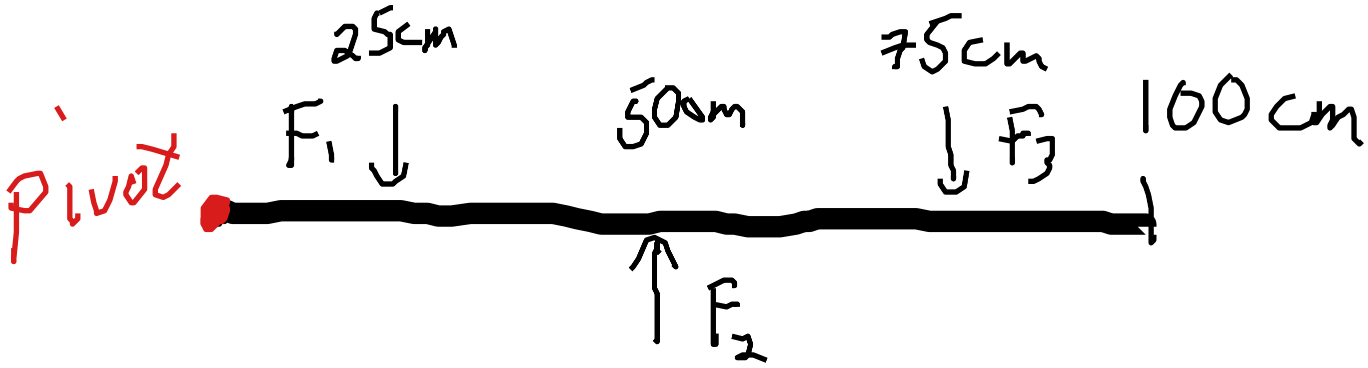


Not in equilibrium
even though $F_{net} = 0$

Static Equilibrium

- $F_{net} = 0$
- $\tau_{net} = 0$
- A stationary object is in static equilibrium



Down = Up

$$\vec{F}_1 + \vec{F}_3 = \vec{F}_2$$

Counterclockwise = Clockwise

(we need to specify a pivot)

$$F_2\left(\frac{1}{2}\right) = F_1\left(\frac{1}{4}\right) + F_3\left(\frac{3}{4}\right)$$

$\tau = r\vec{F}$ when $r \perp F$

r: distance from pivot to application of force

Let

$F_1 = 20$ (for example)

$\rightarrow 20 + F_3 = F_2$

$\frac{1}{2} F_2 = \frac{1}{4}(20) + \frac{3}{4} F_3$

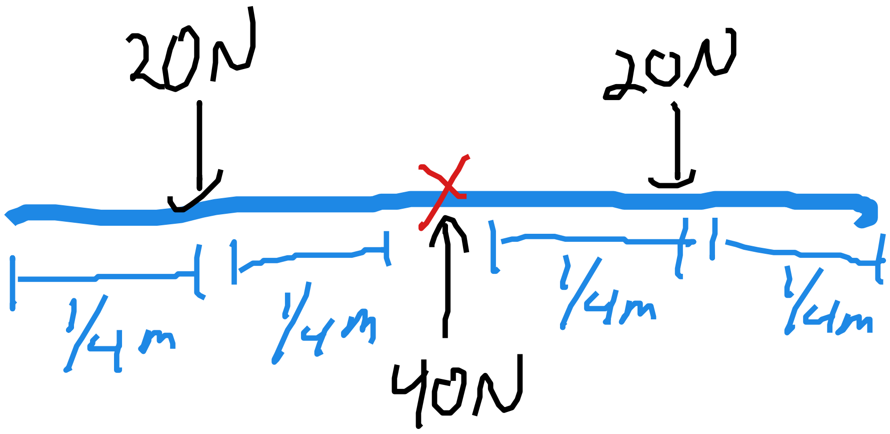
$\frac{1}{2}(20 + F_3) = 5 + \frac{3}{4} F_3$

$10 - 5 = \frac{3}{4} F_3 - \frac{1}{2} F_3 = \frac{1}{4} F_3$

$5 = \frac{1}{4} F_3$

$20 = F_3$

$F_2 = 20 + F_3 = 40N$



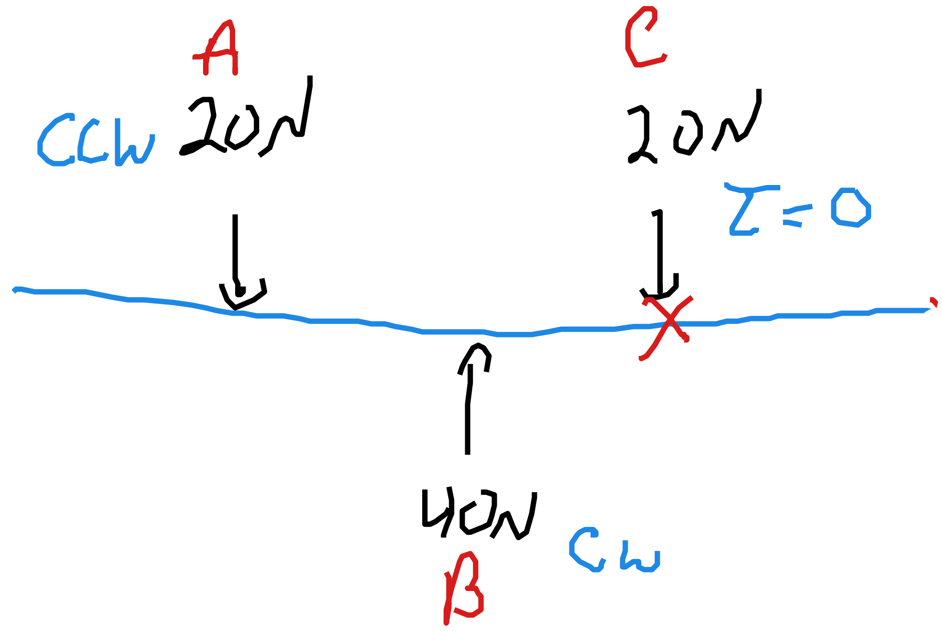
For a system in static equilibrium you can put the pivot anywhere you want

If pivot in middle,

CCW $\quad$ CW

$$20\left(\frac{1}{4}\right) = 20\left(\frac{1}{4}\right)$$

$$5\ Nm = 5\ Nm$$

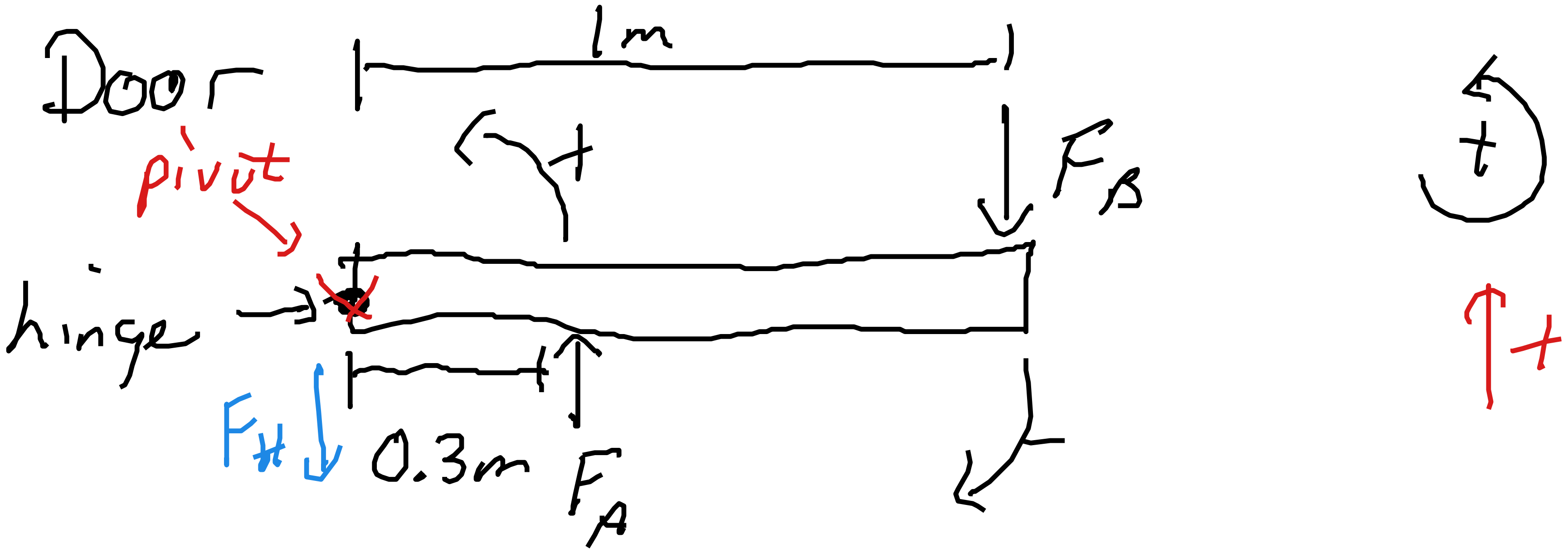


CCW + | CW −

$$20N\left(\tfrac{1}{2}m\right) = (40N)\left(\tfrac{1}{4}m\right) \checkmark$$

$$10Nm = 10Nm$$

$$\tau_{net} = +10Nm - 10Nm = 0$$



If door doesn't move, which force is bigger?

$$\tau_{net} = +F_A(0.3) - F_B(1) = 0$$

$$\Rightarrow 0.3\,F_A = F_B$$

$$F_A > F_B$$

example $\begin{cases} F_A = 10N \\ F_B = 3N \end{cases}$

$$F_{net} = +F_A - F_B + F_H = 0$$

hinge

Hinges apply a force which points in whatever direction they need to to prevent the hinge from being dislocated.

$$F_{net} = 10 - 3 + F_H = 0$$

$$7 + F_H = 0$$

$$F_H = -7$$

If I put the pivot (theoretical point) at the hinge (physical restraint) then the hinge applies no torque but balances out the net force.

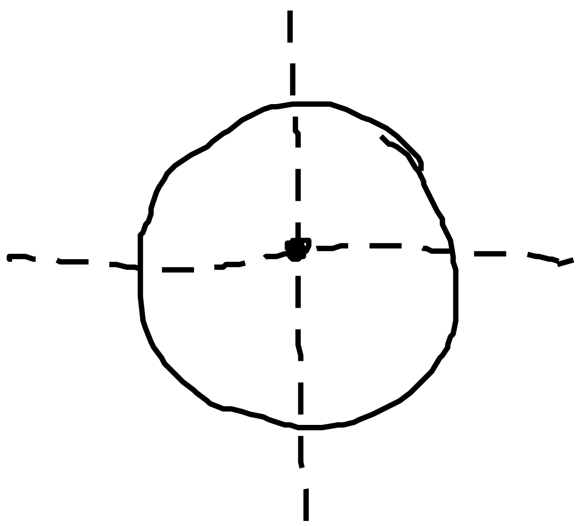
Where do weight and normal force act?

- Weight acts at the center of mass of object.

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$= \cancel{\frac{\int \vec{r}\, dm}{\int dm}}$$

COM lies along any symmetry axis the object has.



If circle's mass is uniformly distributed, then COM is center.

If we place the pivot at COM, then $\tau_{net} = 0$ only if T points directly away from pivot

B: center of base

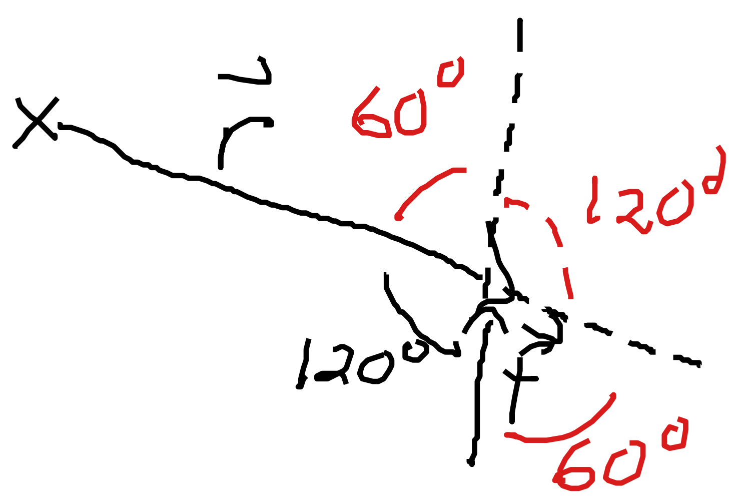
C: below COM

Normal force will move anywhere on the surface of contact it needs to be to maintain equilibrium.

Calculating Torque

$$\tau = rF \sin\phi$$

r: vector from pivot to force's (average) location

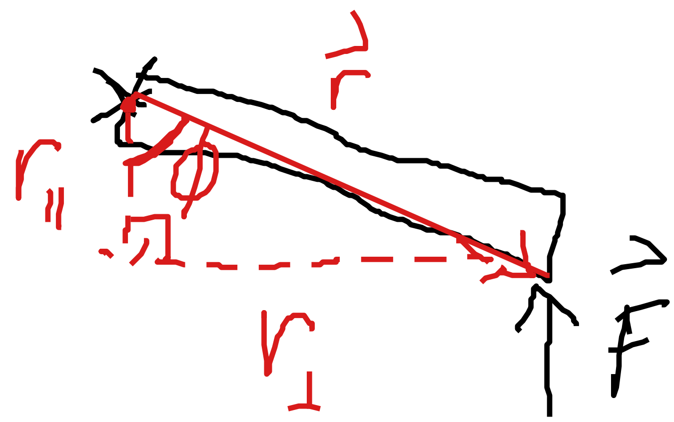


$$\sin(180° - \theta) = \sin\theta$$

$$\sin 120° = \sin 60°$$

use either angle

When F is horizontal or vertical, and r is at an angle



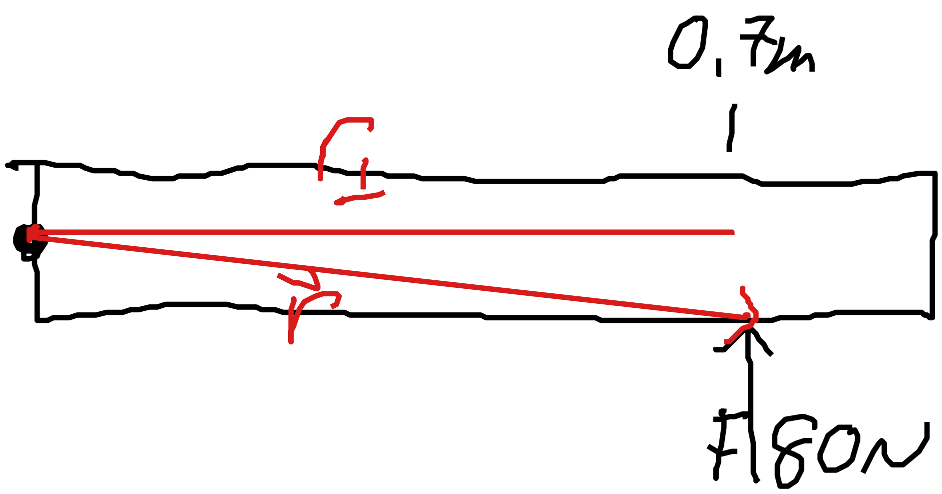
$$\sin\phi = \frac{r_{\perp}}{r}$$

$$\tau = rF\sin\phi$$

$$= rF\frac{r_{\perp}}{r}$$

$$\boxed{\tau = r_{\perp}F}$$

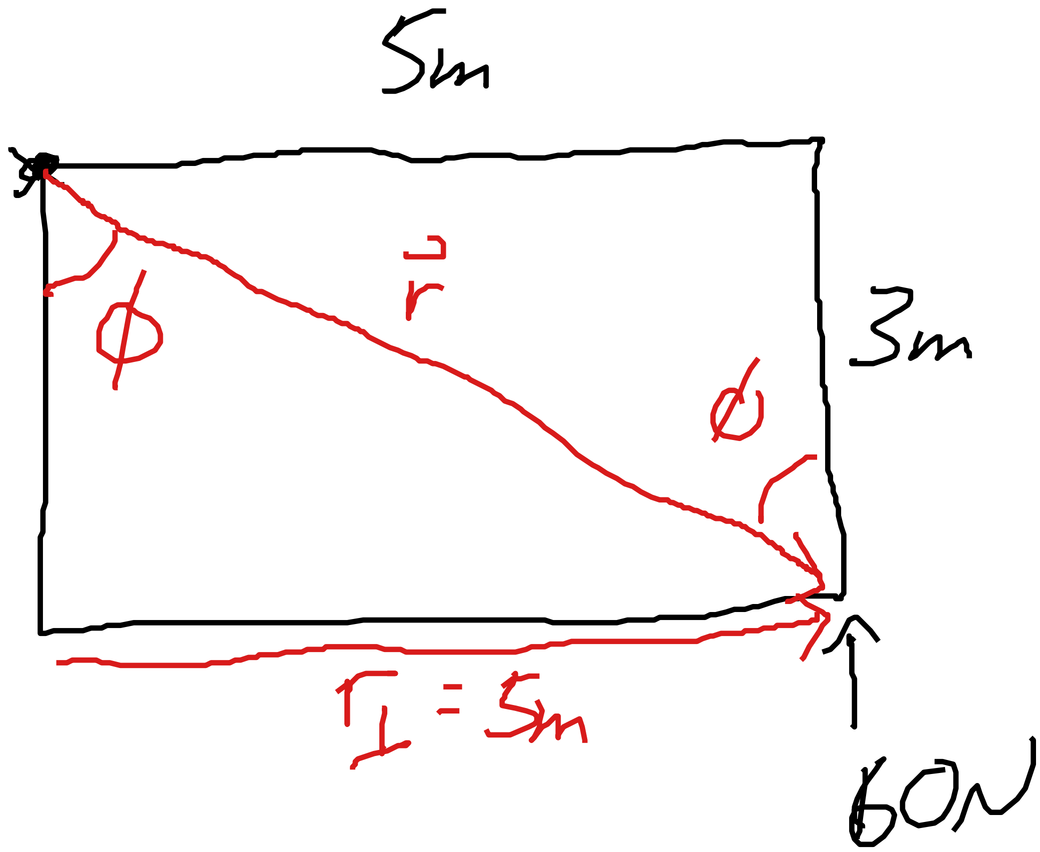
e.g.



$$r_\perp = 0.7\text{m}$$

$$\tau = \overset{r_\perp}{(0.7\text{m})}\overset{F}{(80\text{N})}$$

$$= 56\text{ Nm}$$



$$\tau = rF\sin\phi$$

$$\tau = r_\perp F$$

$$= (5m)(60N) = 300\,Nm$$

$$\boxed{\tau = r F_{\perp}}$$

Useful when r is horizontal or vertical

$\vec{r}$ is horizontal

so $F_{\perp}$ is component of $\vec{T}$ which is vertical

i.e. $8\hat{y}$

$$\tau = rF_{\perp} = (0.5\,\text{m})(8\,\text{N})$$
$$= 4\,\text{Nm}$$

Useful?

$$\tau = rF\sin\phi$$

• when ϕ is given

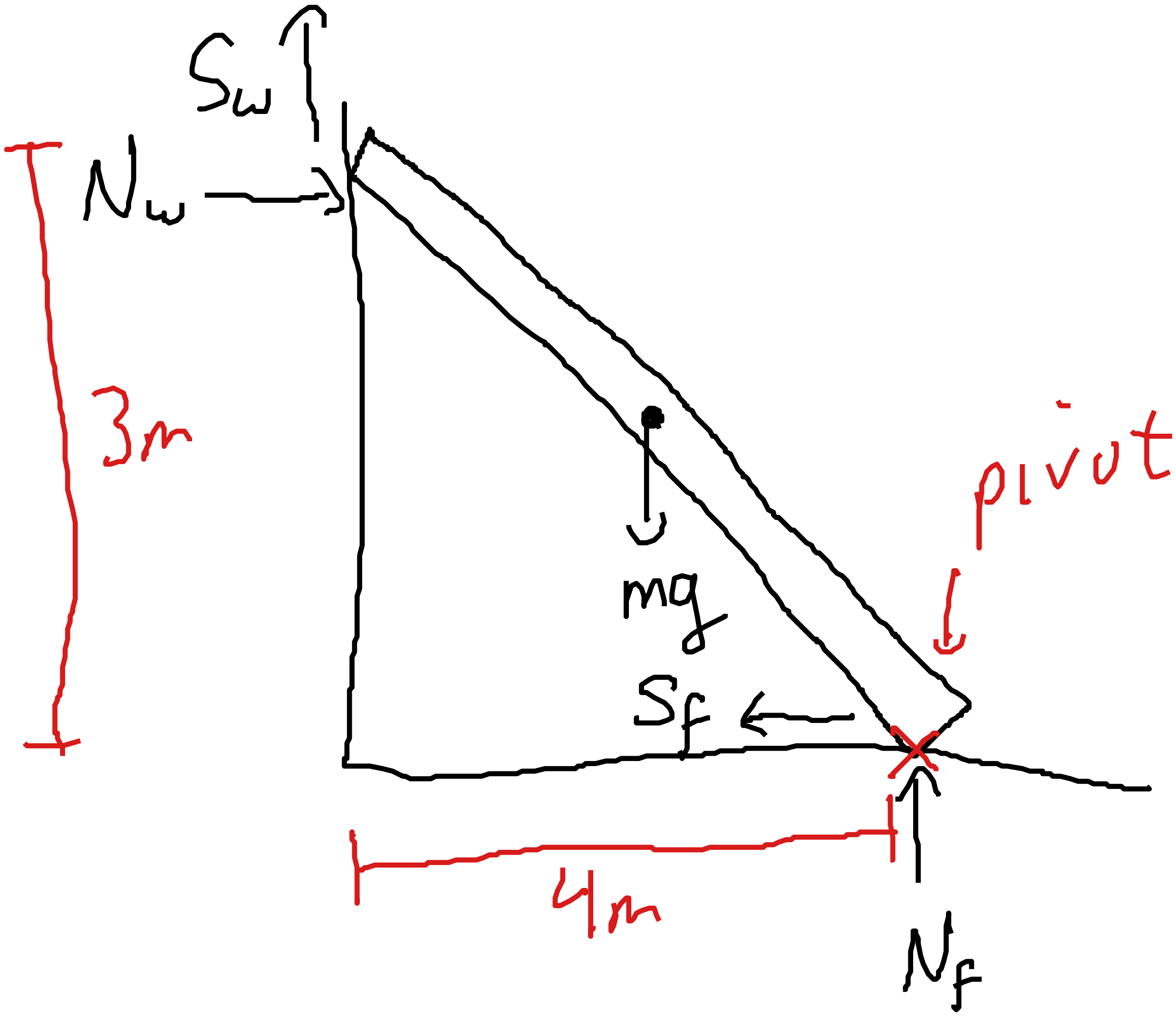
$$\tau = r_\perp F$$

• when $\vec{F}$ lies along $\hat{x}$ or $\hat{y}$

$$\tau = rF_\perp$$

• When $\vec{r}$ lies along $\hat{x}$ or $\hat{y}$

Ladder



Horizontal forces

$$N_w = S_f$$

Vertical forces

$$N_f + S_w = mg$$

Torque

N_f : 0

S_f : 0

mg : $+$ $\quad \tau = r_\perp F$
$= +(2m)(mg)$

S_w : $-$ $\quad \tau = r_\perp F = -(4m) S_w$

N_w : $-$ $\quad \tau = r_\perp F = -(3m) N_w$

$$\tau_{net} = 2mg - 4S_w - 3N_w = 0$$

Four unknowns: N_w, S_w, N_f, S_f

Three equations: x, y, τ

Underdetermined problem

My choice for 4th equation: $S_w = 0$

$mg = 400N$

$N_w = S_f$

$N_f = mg = 400N$

$2mg - 3N_w = 0$

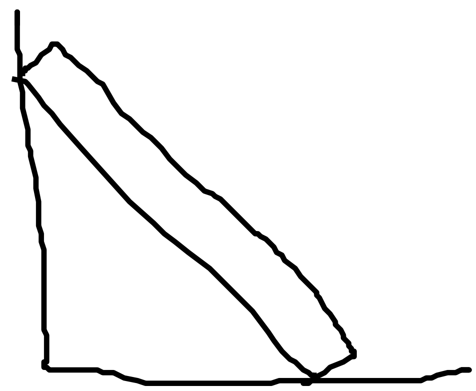
$\to 2mg = 3N_w$

$N_w = \frac{2}{3} mg = \frac{800}{3}$

$= 267N$

$S_f = N_w = 267N$

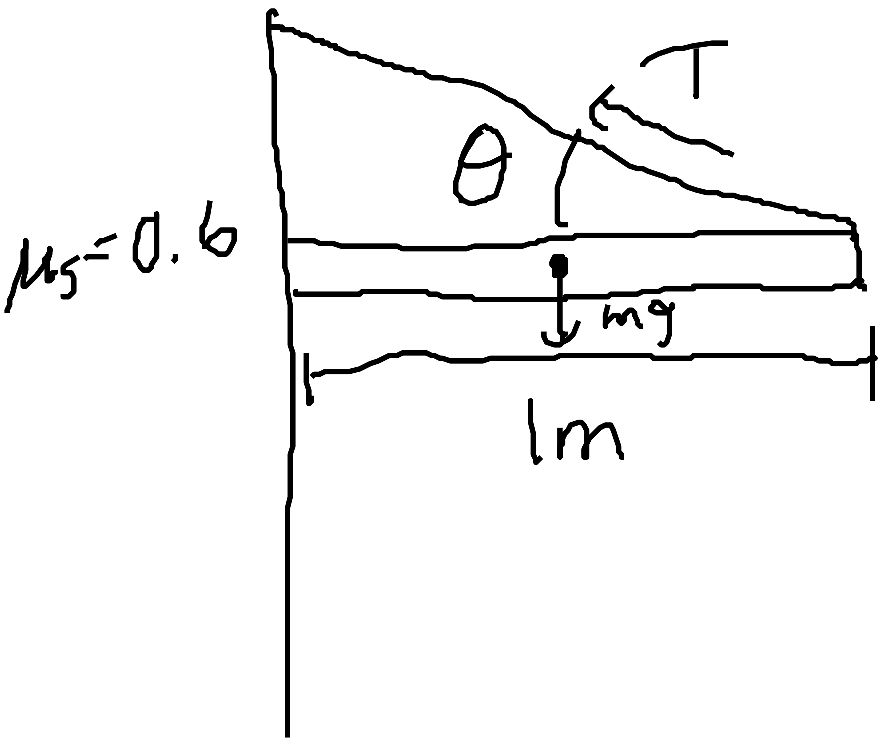
Will the ladder slip?

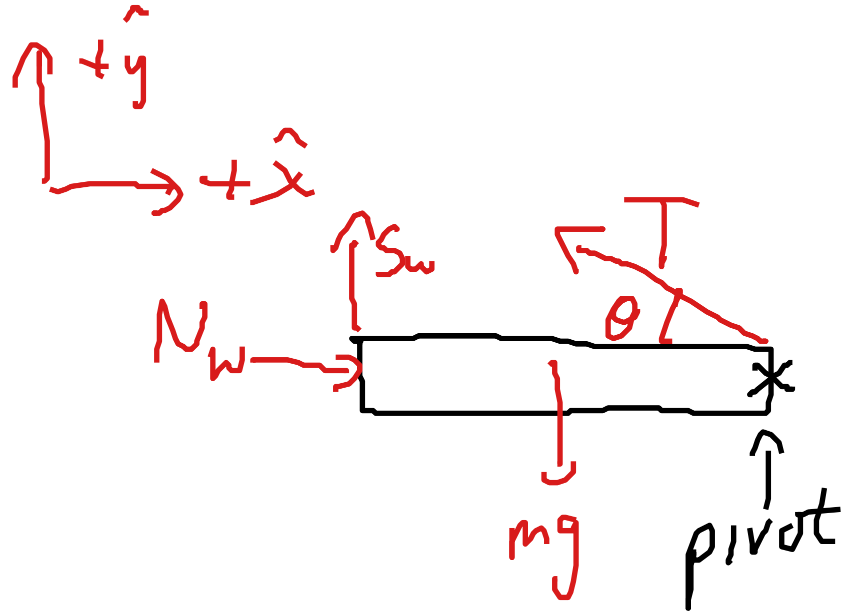


$$S_f \leq \mu_s N_f$$

$$267N \leq \mu_s \, 400N$$

$$\mu_s \geq \frac{267}{400} = 0.89$$





	x	y	τ
Weight	—	$-mg$	$+\frac{1}{2}mg$
Tension, rope	$-T\cos\theta$	$+T\sin\theta$	0
Normal, wall	$+N_w$	—	0
S. friction, wall	—	$+S_w$	$-1S_w$
	0	0	0

x: $N_w = T\cos\theta$

y: $S_w + T\sin\theta = mg$

τ: $\frac{1}{2}mg - S_w = 0$

Unknowns: N_w, S_w, T, θ

4th "equation": $S_w \leq \mu_s N_w$

$$S_w = \frac{1}{2}mg$$

$$\frac{1}{2}mg + T\sin\theta = mg$$

$$T\sin\theta = \frac{1}{2}mg = S_w$$

$$T\cos\theta = N_w$$

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{S_w}{N_w}$$

$$\tan\theta = \frac{S_w}{N_w}$$

$$S_w = N_w \tan\theta$$

$$S_w \leq \mu_s N_w$$

$$\cancel{N_w} \tan\theta \leq \mu_s \cancel{N_w}$$

$$\mu_s \geq \tan\theta$$

$$\theta \leq \tan^{-1} 0.6$$

$$\theta \leq \tan^{-1} \mu_s$$

$$\theta \leq 31^\circ$$