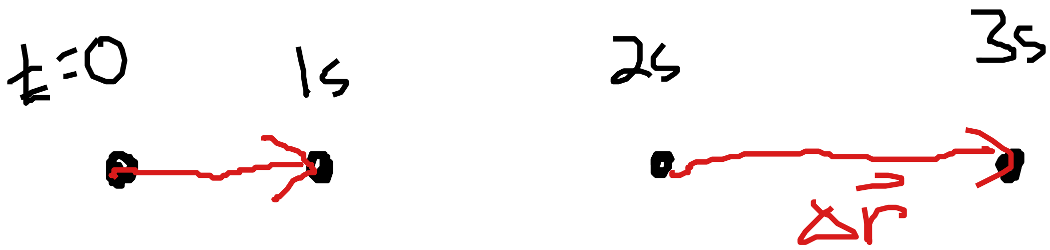
Motion Diagrams



displacement vector

$\Delta\vec{r}$ "delta r-vec"

longer displacement vector = faster motion

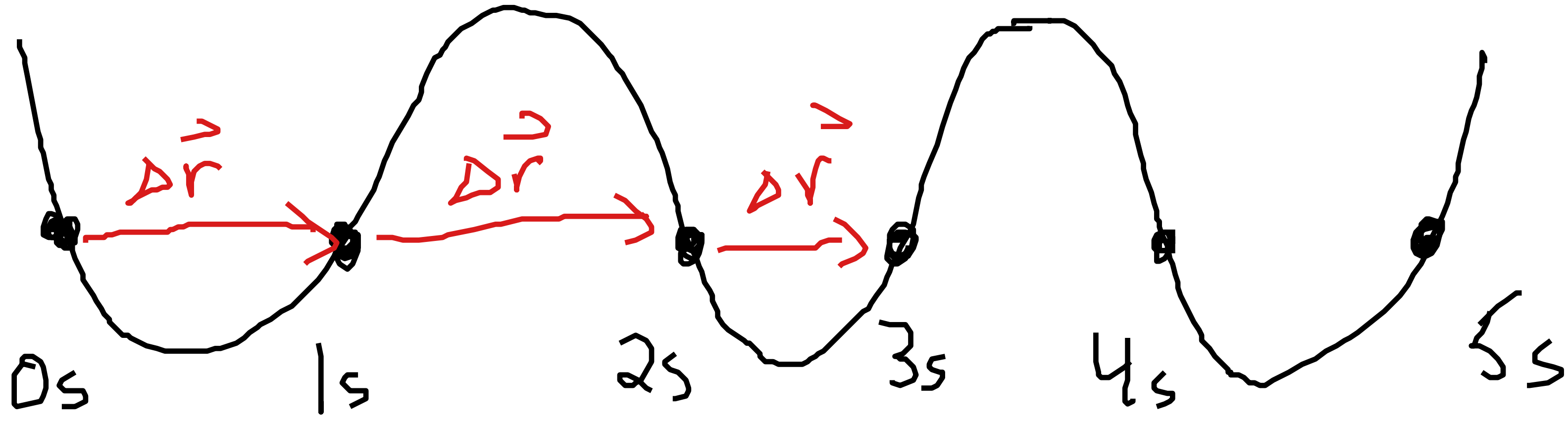
larger time interval Δt = slower motion

Velocity

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

- direction = direction of motion
- magnitude $|\vec{v}| = v$ <u>speed</u>

speed: never negative



$\Delta t = 1s$

$$\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

SI units

meter (m)

kilogram (kg)

second (s)

$1\ m/s \approx 2\ \frac{mi}{hr}$

$\approx 4\ \frac{km}{hr}$

Changes

$$\Delta A = A_f - A_i$$

I start with 5 apples.
An hour later I have
3 apples.

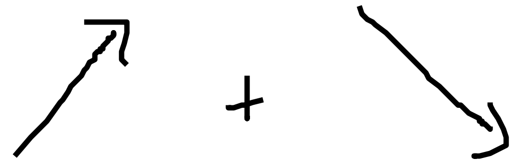
$$\Delta A = 3 - 5 = -2$$

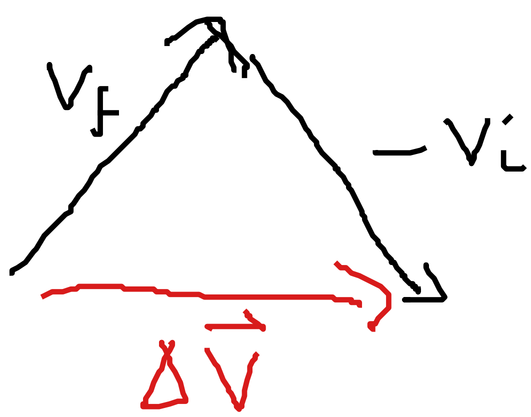
$\to$ -2 apples/hour

rate of change $\dfrac{\Delta A}{\Delta t}$

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i$$

$$= \vec{v}_f + (-\vec{v}_i)$$





$$\vec{v}_f = \vec{v}_i + \Delta \vec{v}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

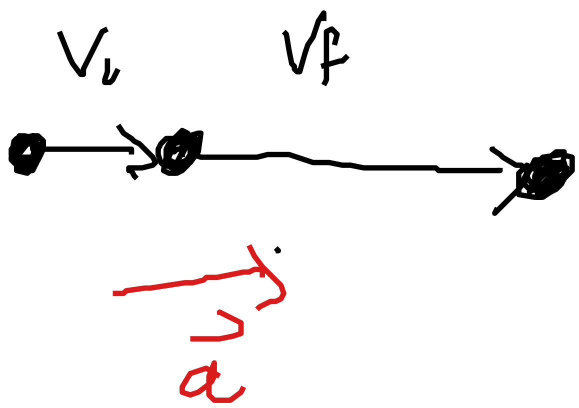
acceleration

Acceleration: rate of change of velocity:

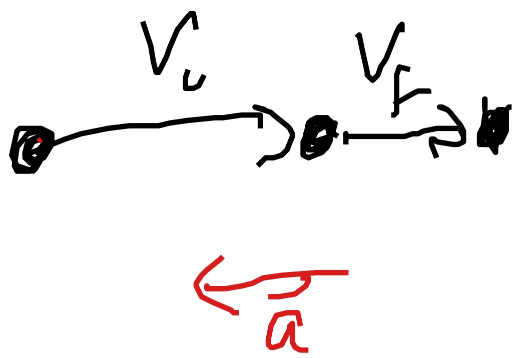
- speed up
- slow down
- change direction

You can't feel velocity

You can feel acceleration as a push in opposite direction

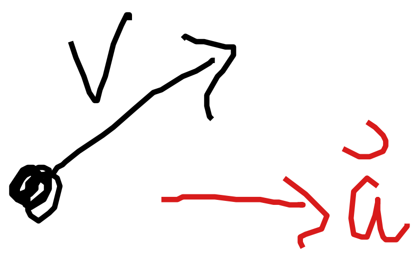


pushed into seat

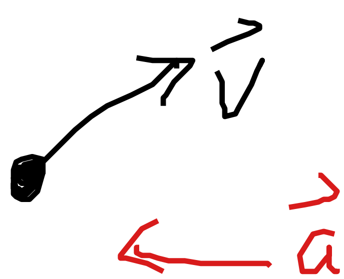


thrown forward

If $\vec{v}$ and $\vec{a}$ are in same-ish direction, speeds up



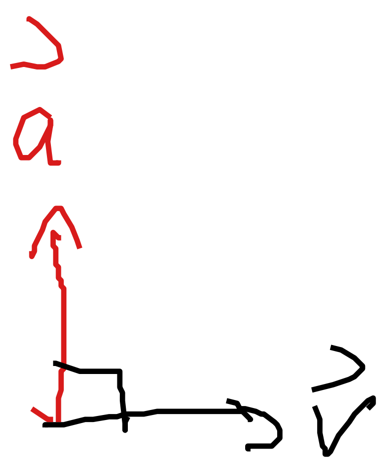
If $\vec{v}$ & $\vec{a}$ are in opposite-ish directions, slows down



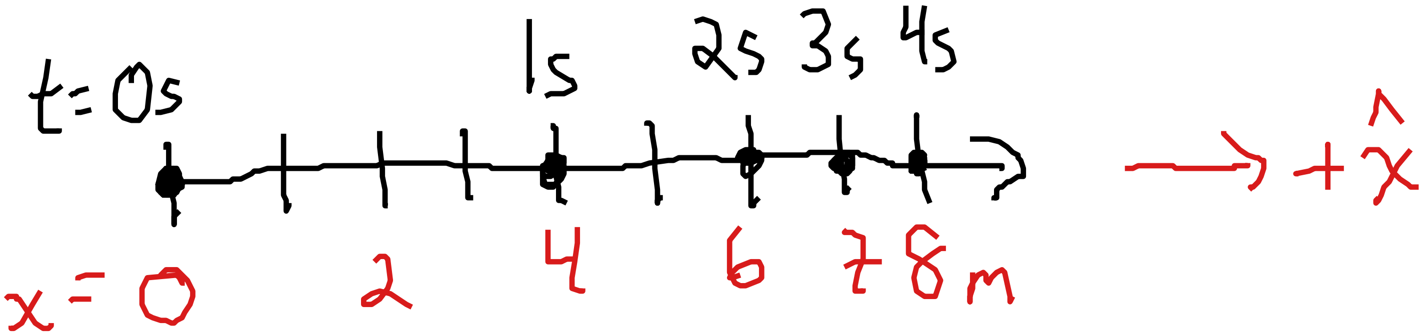
If $\vec{v} \perp \vec{a}$

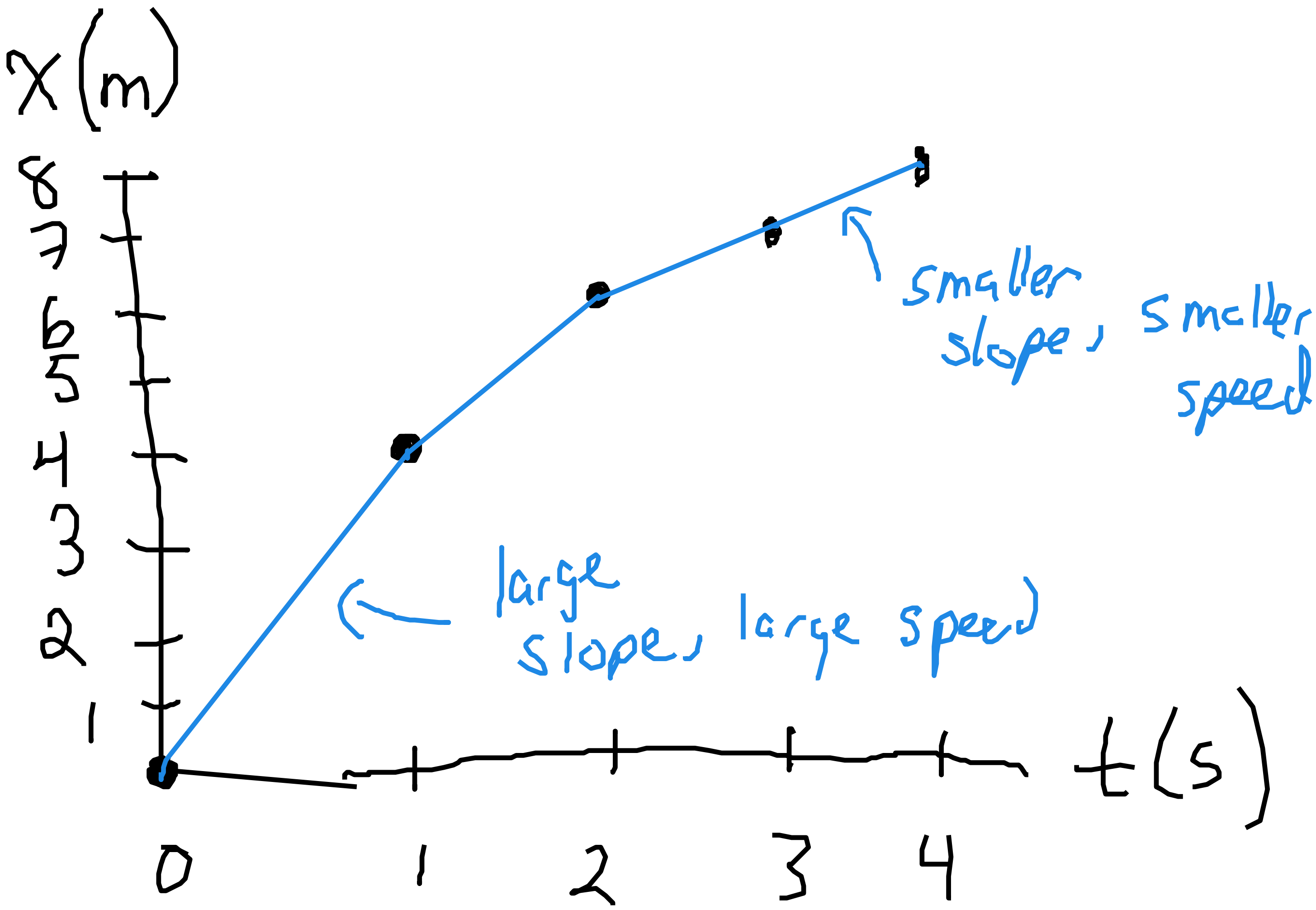
constant speed

changing velocity

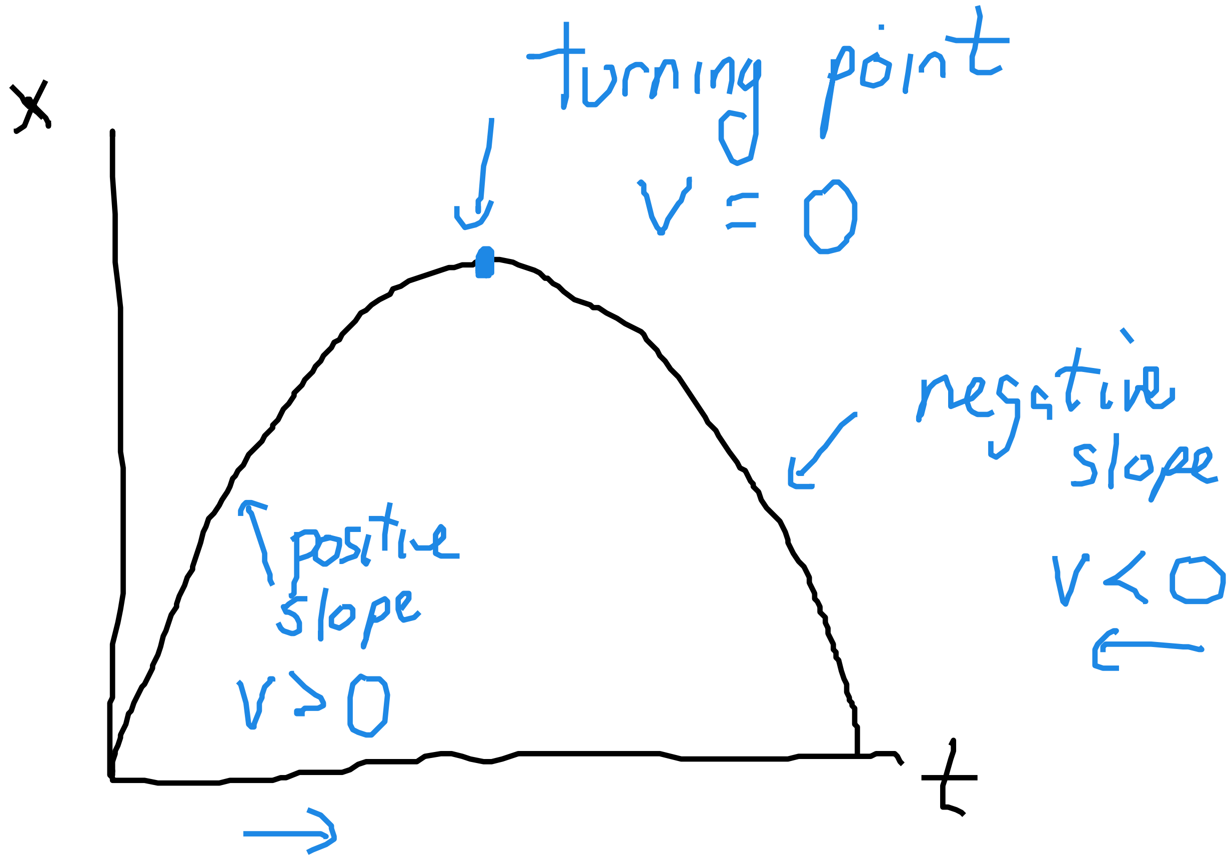


In 1D motion, graphing





$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = v$$

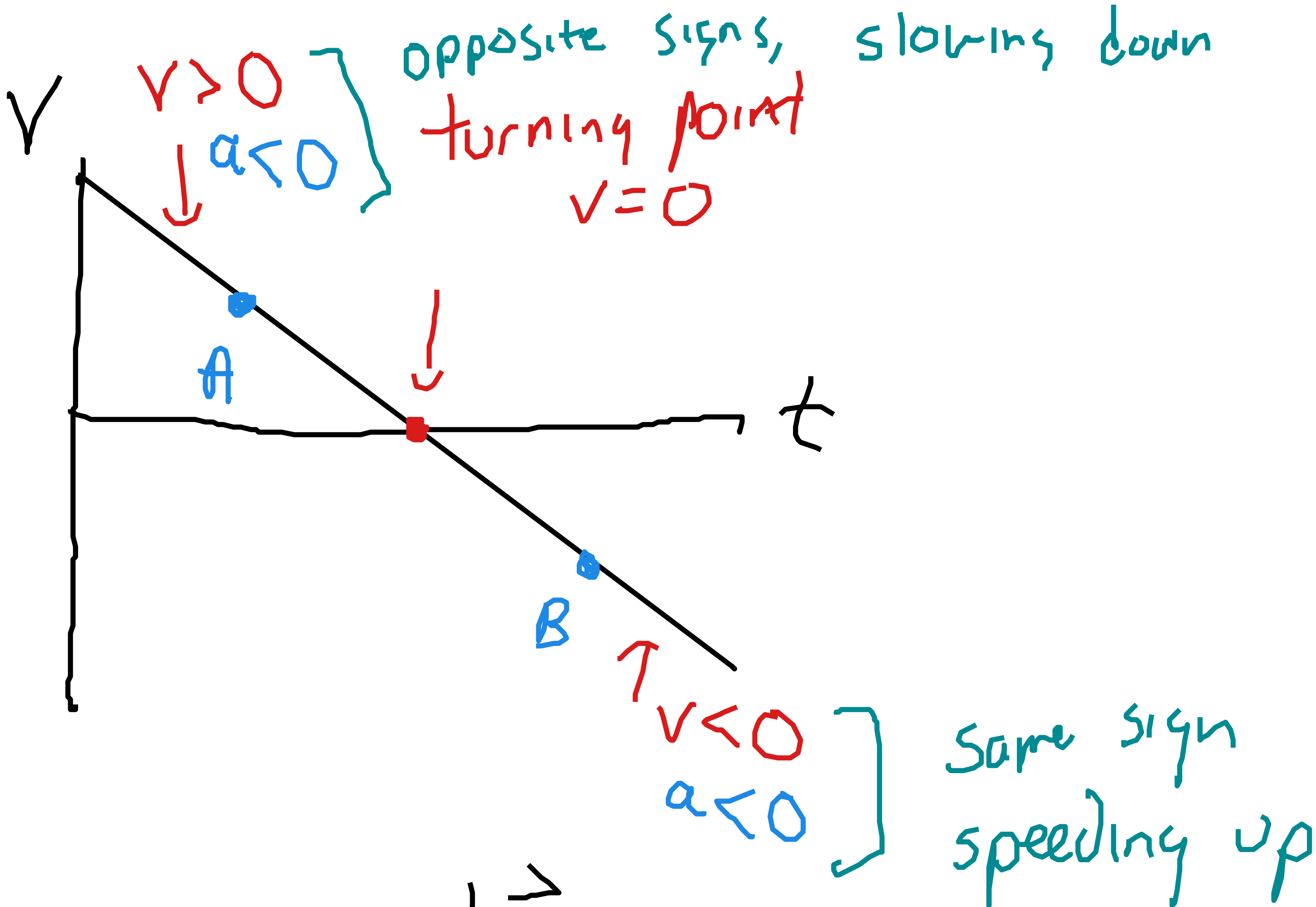


$\longrightarrow +\hat{x}$

slope $\quad \vec{v} = \dfrac{d\vec{x}}{dt}$

e.g. $\quad x(t) = 9 - (t-3)^2$

$$v = \frac{dx}{dt} = -2(t-3) = -2t + 6$$



$$\vec{a} = \frac{d\vec{v}}{dt}$$

Acceleration at turning point
is negative (not zero)

When is object speeding up?

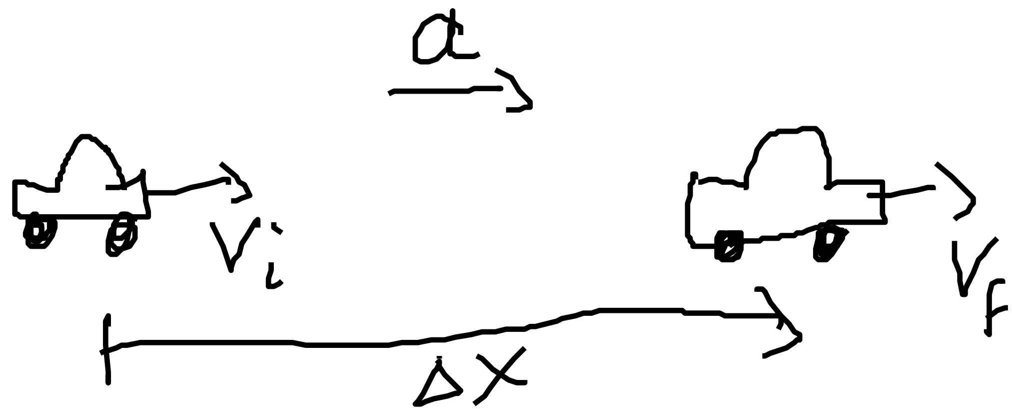
A B Both Neither

If v & a have same sign,
speeding up

If v & a have opposite signs,
slowing down

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{v} = \frac{d\vec{x}}{dt}$$

Constant Acceleration in 1D



- acceleration a
- initial velocity v_i
- final velocity v_f
- duration of motion Δt
- displacement Δx or Δy

We need to know $5 - 2 = \boxed{3}$ of these to solve

$$a = \frac{dv}{dt}$$

$$\rightarrow \int_i^f dv = \int_i^f a\, dt$$

$$= [v]_i^f = a[t]_i^f$$

$$= v_f - v_i = a(t_f - t_i)$$

$$v_f - v_i = a\Delta t$$

$$\boxed{v_f = v_i + a\Delta t}$$

$$v = \frac{dx}{dt}$$

$$v(t) = v_i + at$$

$$\int_i^f dx = \int_i^f v \, dt$$

$$x_f - x_i = \int_i^f (v_i + at) \, dt$$

$$= \int_i^f v_i \, dt + \int_i^f at \, dt$$

$$= v_i \int_i^f dt + a \int_i^f t \, dt$$

$$= v_i \Delta t + a \frac{1}{2} \left[t^2 \right]_i^f$$

$$\boxed{\Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2}$$

$$v_f = v_i + a\Delta t \quad \text{(no } \Delta x\text{)}$$

$$\Delta x = v_i \Delta t + \frac{1}{2}a(\Delta t)^2 \quad \text{(no } v_f\text{)}$$

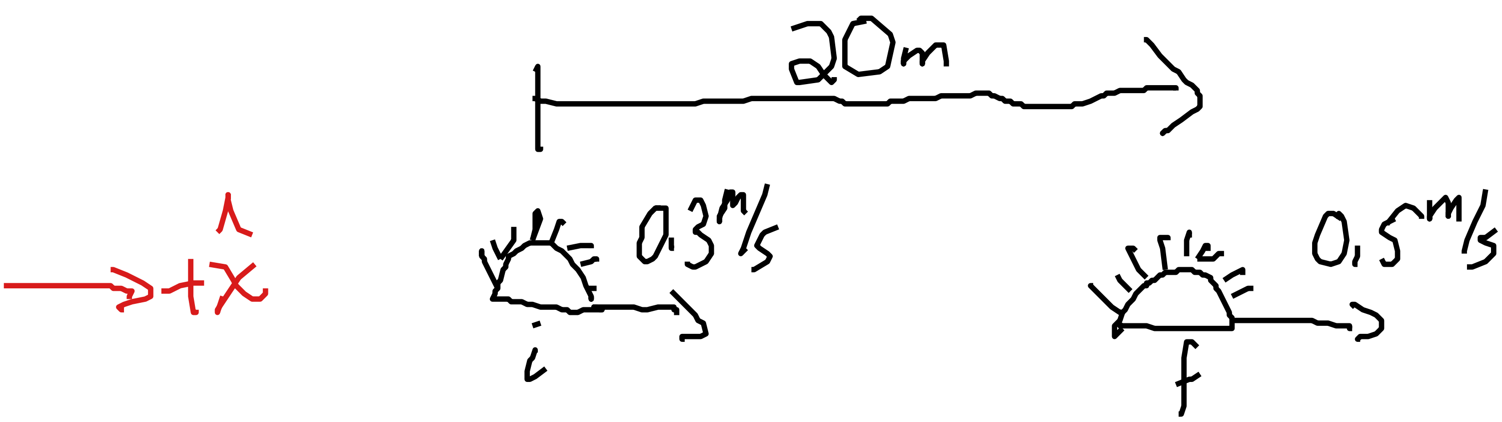
$$\Delta x = v_f \Delta t - \frac{1}{2}a(\Delta t)^2 \quad \text{(no } v_i\text{)}$$

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t \quad \text{(no } a\text{)}$$

$$v_f^2 = v_i^2 + 2a\Delta x \quad \text{(no } \Delta t\text{)}$$

5 equations
(2 independent)

e.g. A Porcupine moving with constant acceleration.
Initially it walks at 0.3 m/s →.
After it travels 20m, it is walking at 0.5 m/s →.
What is its acceleration?



$\Delta x = +20$

$v_i = +0.3$

$v_f = +0.5$

a : NEED

Δt Don't Know, Don't Care

DKDC

Choose equation without Δt

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$(0.5)^2 = (0.3)^2 + 2a(20)$$

$$0.25 - 0.09 = 40a$$

$$a = \frac{0.16}{40} = 0.004\ m/s^2$$

Units of $a = \frac{\Delta v}{\Delta t}$

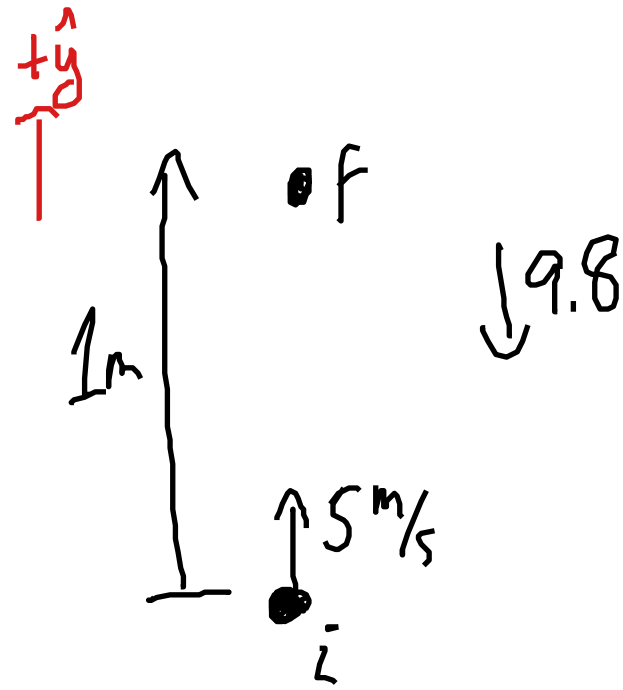
$$\frac{m/s}{s} = \frac{m}{s \cdot s} = m/s^2$$

Acceleration due to gravity

All objects in free fall accelerate downward at $9.8\ m/s^2$

$$\vec{a} = g \downarrow \qquad g = 9.8\ m/s^2$$

I throw a ball in the air at 5 m/s. How long does it take to be 1m above my hand?



Δy : +1

V_i : +5

V_f : DKDC

a : −9.8

Δt : NEED

$$\Delta y = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$1 = 5(\Delta t) + \frac{1}{2}(-9.8)(\Delta t)^2$$

$$ax^2 + bx + c = 0$$

$$4.9(\Delta t)^2 - 5(\Delta t) + 1 = 0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4.9)(1)}}{2(4.9)}$$

$$= \frac{5}{9.8} \pm \frac{\sqrt{5.4}}{9.8}$$

$$= \frac{5 \pm 2.32}{9.8} = 0.27s \text{ or } 0.75s$$

At 0.27s it is 1m above my hand & moving up.

At 0.75s it is 1m above my hand & moving down.

Throw ball up at $5\ m/s$.
How high does it go?

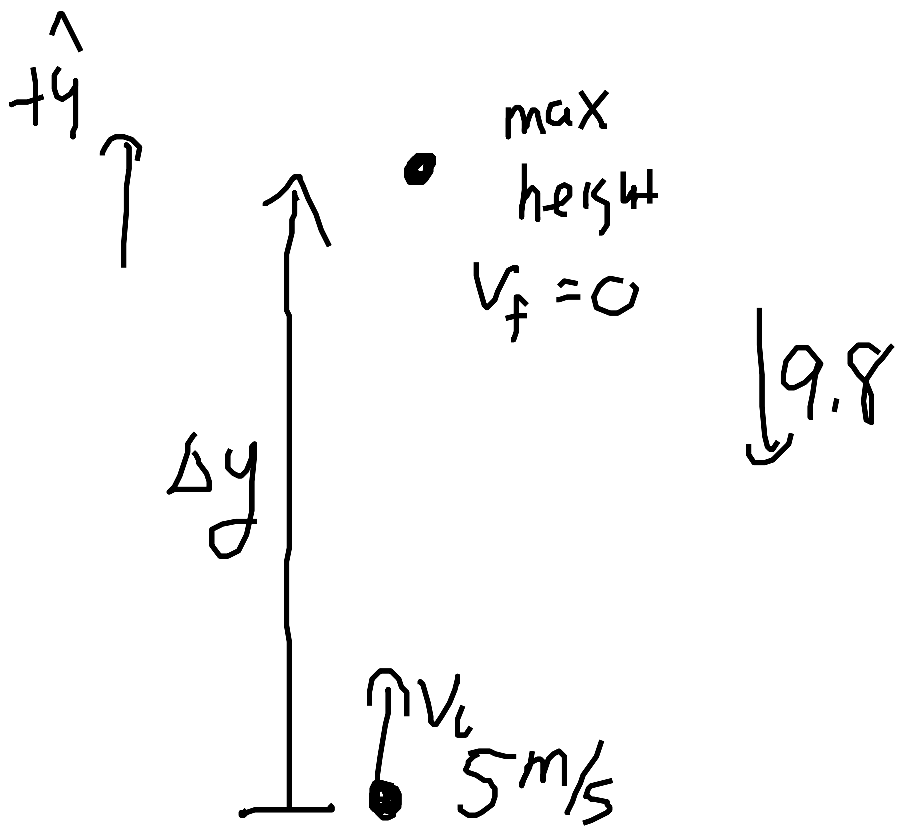
Δy = NEED

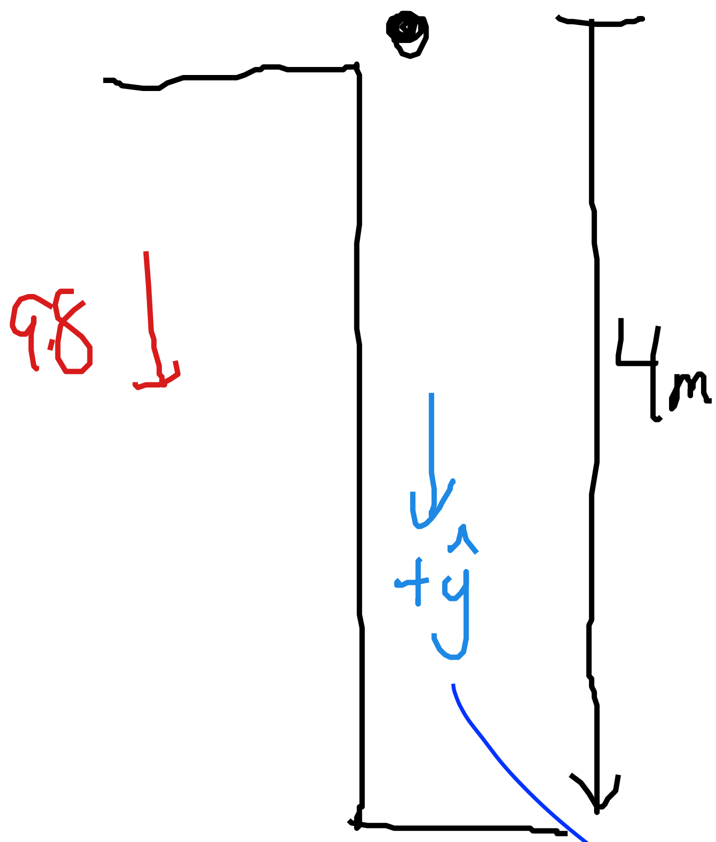
$v_i = +5$

$v_f = 0$

$a = -9.8$

Δt = DKDC





A ball is dropped from the top of a 4m building

How fast is it moving when it reaches the ground?

NOTE: I decided to make down "positive" in this problem, which is why a is +9.8 and Δy is +4. If you made up "positive" as it usually is, then both values would be negative.

$\Delta y = +4$

$v_i = 0$

v_f

$a = +9.8$

Δt