Constant Acceleration in 1D Problems

Five variables: $\overbrace{\Delta x,\ V_i,\ V_f,\ a}^{\text{vectors: sign = direction}},\ \Delta t$

(Δx or Δy)

Need 3 of these

$$V_f = V_i + a\Delta t \qquad \text{(no } \Delta x\text{)}$$

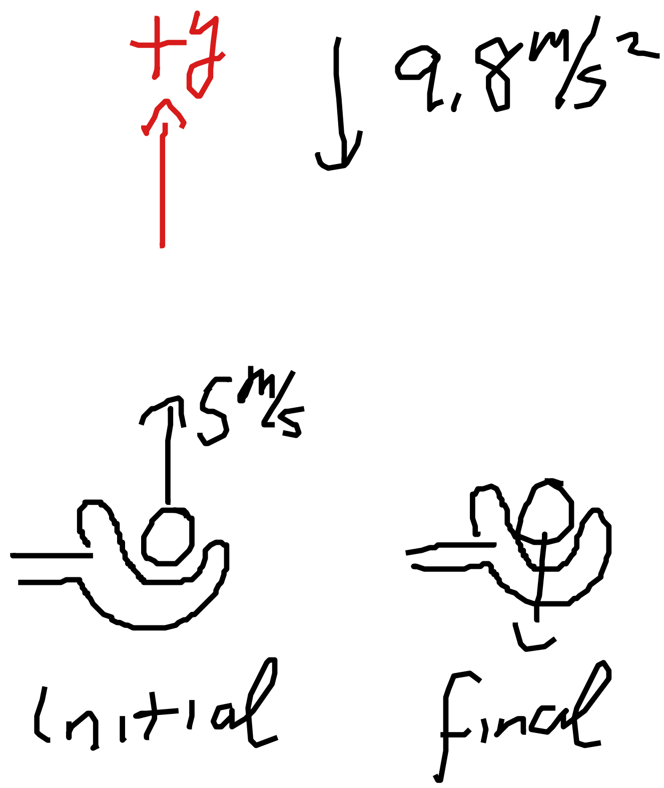
$$\Delta x = V_i \Delta t + \frac{1}{2}a(\Delta t)^2 \qquad \text{(no } V_f\text{)}$$

$$\Delta x = V_f \Delta t - \frac{1}{2}a(\Delta t)^2 \qquad \text{(no } V_i\text{)}$$

$$V_f^2 = V_i^2 + 2a(\Delta x) \qquad \text{(no } \Delta t\text{)}$$

$$\Delta x = \frac{1}{2}(V_i + V_f)\Delta t \qquad \text{(no } a\text{)}$$

I throw a ball into the air at 5 m/s. How fast is it moving when it hits my hand again?



Δx 0

V_i $+5\ m/s$

V_f NEED

a $-9.8\ m/s^2$

Δt DKDC

$$V_f^2 = V_i^2 + 2a(\Delta x)$$

$$V_f^2 = (5)^2 + 0$$

$$V_f^2 = 25 \rightarrow V_f = \pm 5\ m/s$$

Answer is $v_f = -5\,m/s$

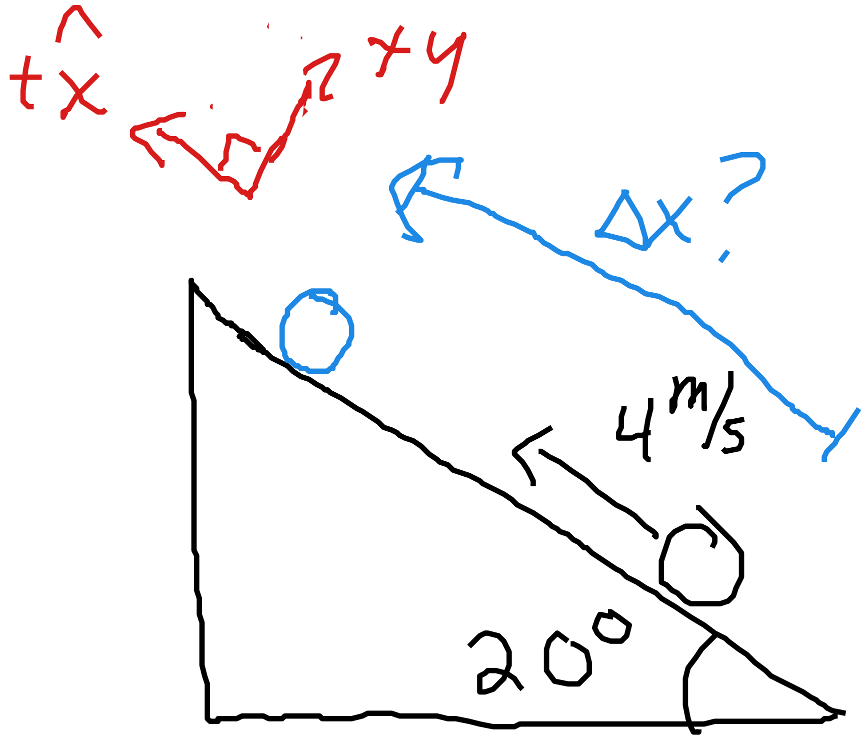
because ball is

moving down when it

returns to my hand.

The $+5\,m/s$ solution

corresponds to $\Delta t = 0$.



How far does the ball go before it stops? (No friction)

Δx NEED

v_i $+4$

v_f 0

a $-9.8 \sin 20^\circ$

Δt DKDC

$a = 0$

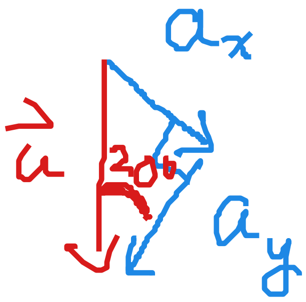
$= -9.8$

$= -9.8 \sin 20^\circ$

$= -9.8 \cos 20^\circ$

$= -9.8 \tan 20^\circ$

$$\boxed{\begin{aligned} \sin 0^\circ &= 0 \\ \cos 0^\circ &= 1 \end{aligned}}$$



$$a_x = -9.8 \underset{\cos}{\sin} 20^\circ$$

When you have an incline of θ, : x component of gravity is $\pm g \sin\theta$

& y component is $\pm g \cos\theta$.

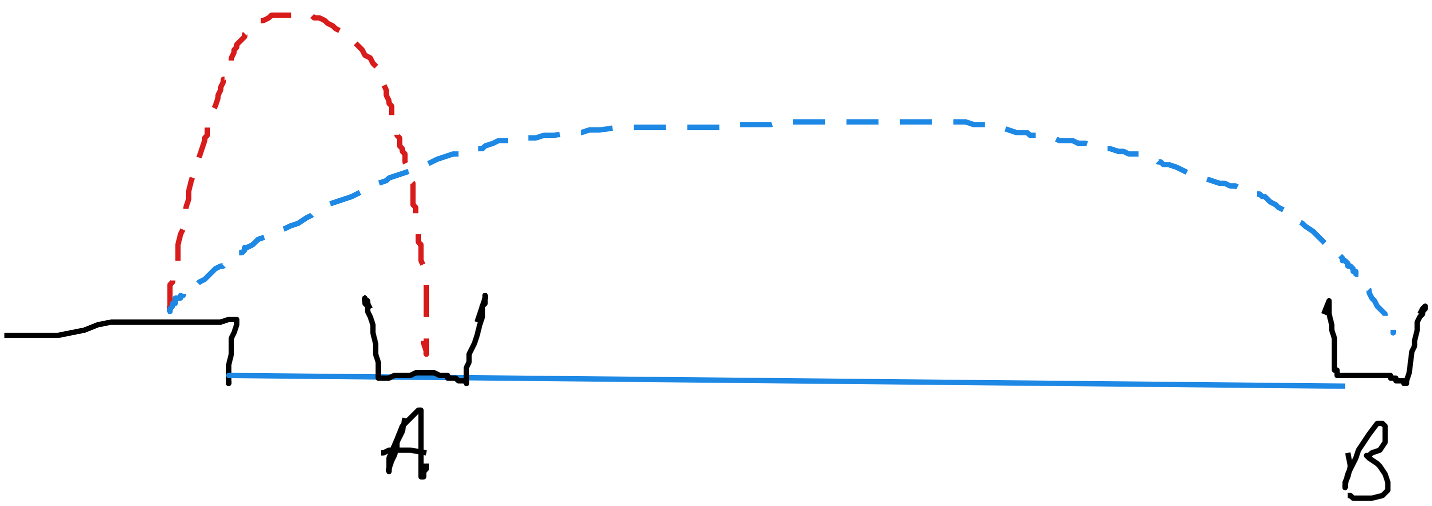
$$v_f^2 = v_i^2 + 2a(\Delta x)$$

$$0 = 4^2 + 2(-9.8\sin 20^\circ)\Delta x$$

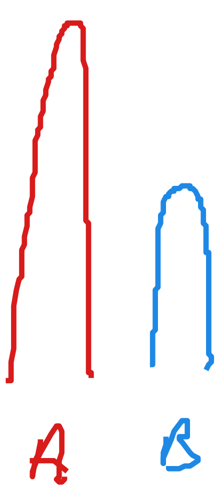
$$\frac{16}{19.6\sin 20^\circ} = \Delta x = 2.39\,m$$

2D Kinematics

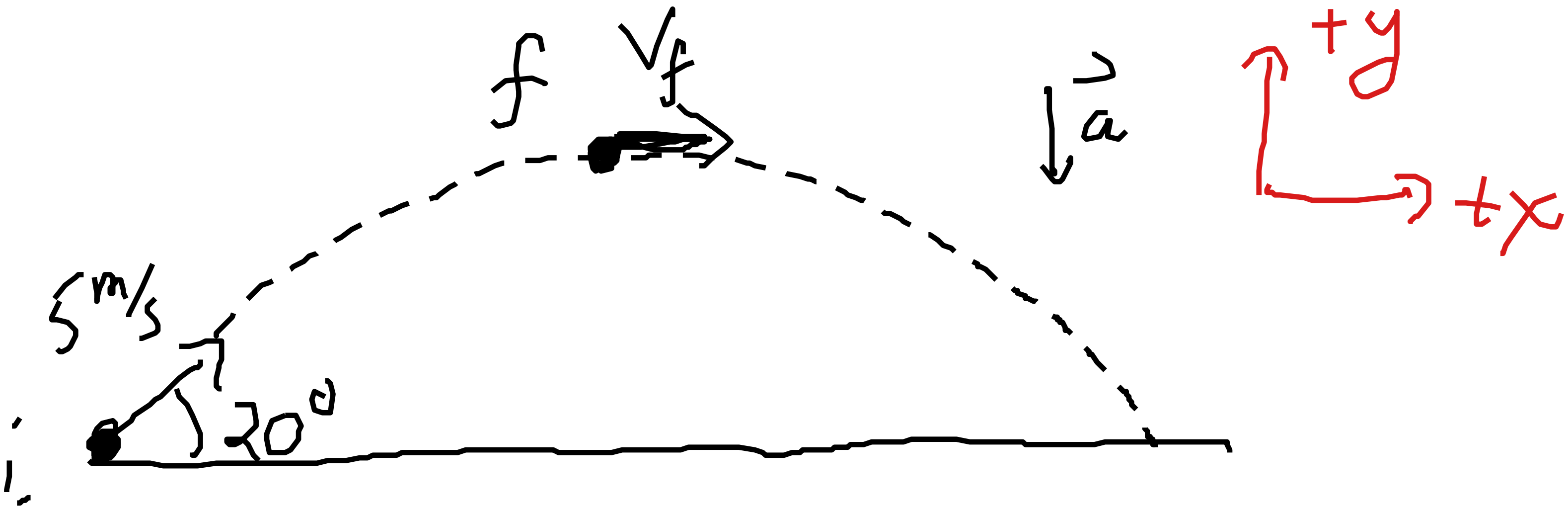
- x & y are independent of each other



Both cannonballs are launched at same time. Which boat gets hit first?



B hits first because vertically it has less for to go.



How long until the ball reaches its highest point?

Δx ? Δy ?

V_{ix}: $+5\cos 20°$ V_{iy}: $+5\sin 20°$

V_{fx}: ? V_{fy}: 0

a_x 0 a_y: -9.8

Angle measured from x, so x gets the cos.

Δt NEED

Time belongs to both columns.
We need 3 knowns in a column to solve for it.

Y column has 3 knowns so use it to solve for Δt

$$V_{fy} = V_{iy} + a_y \Delta t$$

$$0 = 5 \sin 20^\circ + (-9.8) \Delta t$$

$$\Delta t = \frac{5 \sin 20^\circ}{9.8} = 0.175 s$$

But wait isn't $V_{fx} = 5 \cos 20^\circ$ too?

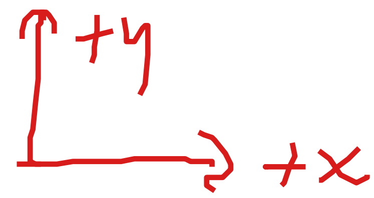
Since $a_x = 0$?

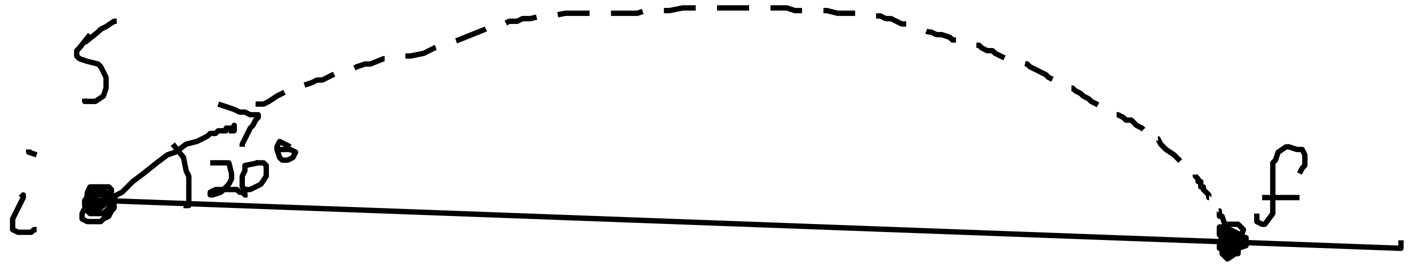
$$V_{fx} = V_{ix} + a_x \Delta t$$

$$5\cos 20^\circ = 5\cos 20^\circ + 0(\Delta t)$$

$$5\cos 20^\circ = 5\cos 20^\circ$$

oops no Δt!





Wind blows which gives a horizontal acceleration of $2\,m/s^2 \leftarrow$

How for away does the ball land?

Δx = NEED $\qquad \Delta y = 0$

V_{ix} $+5\cos 20^\circ$ $\qquad V_{iy}$ $+5\sin 20^\circ$

V_{fx} ? $\qquad V_{fy}$?

a_x -2 $\qquad a_y$ -9.8

Δt

- We need to solve x, but only 2 givens.
- Solve y column for Δt, then solve x.

$$\Delta y = V_{iy}\,\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$0 = (5\sin 20^\circ)\,\Delta t + \frac{1}{2}(-9.8)(\Delta t)^2$$

$$\frac{4.9(\Delta t)^2}{\Delta t} = \frac{1.71\,\Delta t}{\Delta t} \quad (\Delta t \neq 0)$$

$$4.9\,\Delta t = 1.71$$

$$\Delta t = 0.349\,s$$

$$\Delta x = v_{ix}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2$$

$$\Delta x = (5\cos 20^\circ)(0.349) + \frac{1}{2}(-2)(0.349)^2$$

$$= 1.52\,m$$

$$\frac{d}{dx}\left(x^a\right) = ax^{a-1}$$

$$\frac{d}{dx}\left(x^2\right) = 2x$$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(x^1\right) = 1x^0$$

$$= 1$$

$$\frac{d}{dx}(3) = 0.$$

Derivative Review

$$\frac{d}{dx}\left(3x^2 + 2x^5\right)$$

$$\frac{d}{dx}\left(3x^2\right) + \frac{d}{dx}\left(2x^5\right)$$

$$3\frac{d}{dx}x^2 + 2\frac{d}{dx}x^5$$

$$3(2x) + 2(5x^4)$$

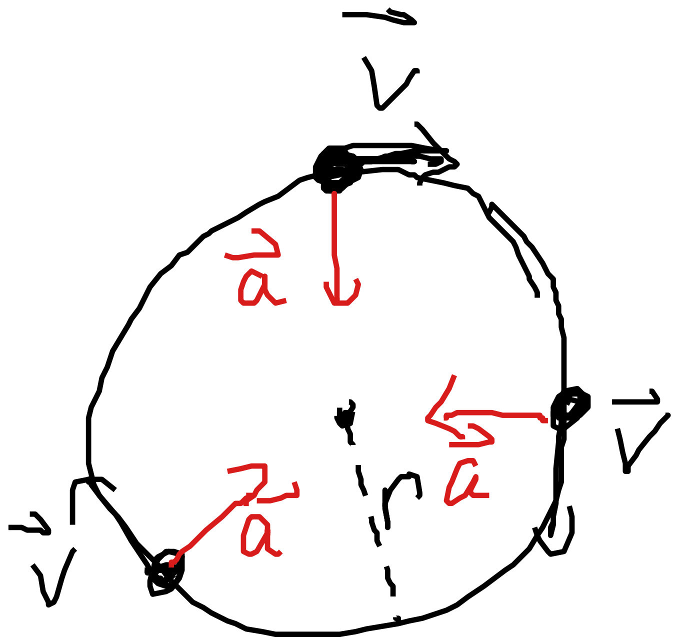
$$6x + 10x^4$$

$$v(t) = \frac{dx(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt}$$

Uniform Circular Motion

(speed is constant)



Centripetal Acceleration

"center" "seeking"

$$\boxed{\vec{a} = \frac{v^2}{r} \text{ towards center} \quad \text{if } v \text{ constant}}$$

Merry-go-round

$r = 2\,m$

$v = 1\,m/s$ walking pace

$$a = \frac{v^2}{r} = \frac{(1)^2}{2} = 0.5\,m/s^2$$

How fast must v be to get $a = g$?

$$9.8 = \frac{v^2}{2} \Rightarrow v^2 = 19.6$$

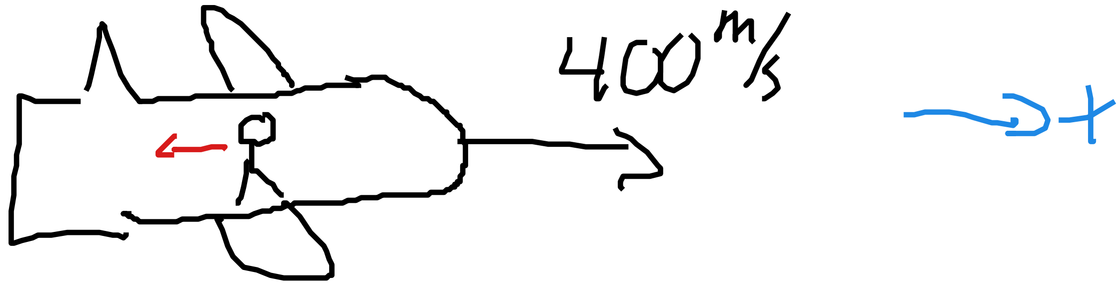
$$v = \sqrt{19.6} = 4.4\,m/s$$

$$= 9\,mph$$

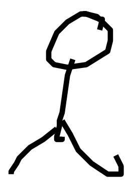
Relative Velocity

- Galilean relativity

  physics is same regardless of how fast you're moving so long as you're not accelerating



walking speed is 1 m/s.



In airplane frame of reference person is moving 1 m/s ←

In outside frame, person is moving 399 m/s →

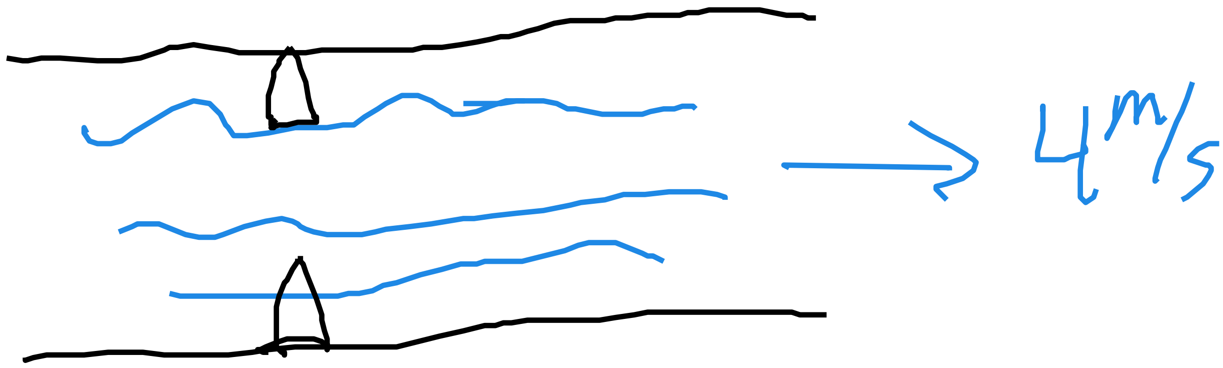
$$\vec{V}_{pg} = +400\ m/s$$

velocity of plane with respect to the ground

$$\vec{V}_{wp} = -1\ m/s$$

$$\vec{V}_{wg} = \vec{V}_{wp} + \vec{V}_{pg}$$

$$+399 = -1 + 400$$



I want the boat to end up directly across the river.

r: river

b: boat

g: ground

$\vec{V}_{bg} = \uparrow$

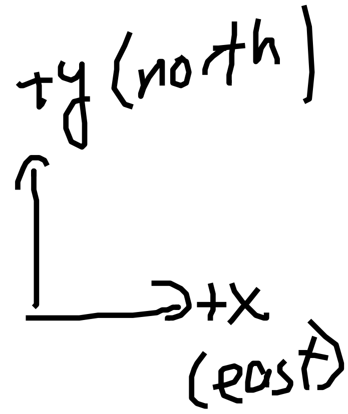
$\vec{V}_{rg} = \longrightarrow$

$\vec{V}_{gr} = -\vec{V}_{rg}$

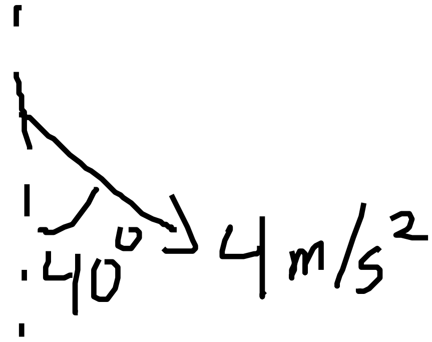
$$\vec{V}_{br} = \vec{V}_{bg} + \vec{V}_{gr}$$

$$\uparrow + \leftarrow = \nwarrow$$

Boat on Water



wind blowing $\vec{a}$



- How fast is boat moving after 5 seconds?
- How far away has boat gone in 5 seconds?

Δx NEED (2) $\quad \Delta y$ NEED (2)

$V_{ix} = 0 \quad V_{iy} = +4$

V_{fx} NEED (1) $\quad V_{fy}$ NEED (1)

$a_x = +4 \sin 40^\circ \quad a_y = -4 \cos 40^\circ$

$\Delta t = 5$

1) $V_{fx} = V_{ix} + a_x \Delta t \qquad V_{fy} = V_{iy} + a_y \Delta t$

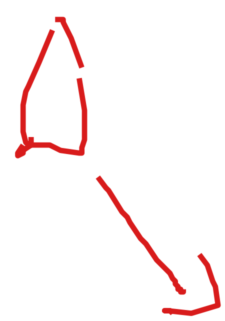
$V_{fx} = (0) + (4 \sin 40^\circ)(5) \qquad V_{fy} = 4 + (-4 \cos 40^\circ)(5)$

$= +12.9 \text{ m/s} \qquad = -11.3 \text{ m/s}$

$|\vec{V}_f| = |+12.9\hat{x} - 11.3\hat{y}|$

$|\vec{V}_f| = \sqrt{(12.9)^2 + (-11.3)^2}$

$= 17.1 \text{ m/s}$



$$\Delta x = \frac{1}{2}(v_{ix} + v_{fx})\Delta t$$

$$= \frac{1}{2}(0 + 12.9)(5)$$

$$= 32.2\,m$$

$$\Delta y = \frac{1}{2}(v_{iy} + v_{fy})\Delta t$$

$$= \frac{1}{2}(4 - 11.3)(5)$$

$$= -18.2\,m$$

$$|\Delta \vec{x}| = \sqrt{(32.2)^2 + (18.2)^2}$$

$$= 37\,m$$

$$\cos 40^\circ = \frac{adj}{hyp}$$

$$= \frac{V_y}{V}$$

$$V_y = -V\cos 40^\circ$$

$$\cos 50^\circ = \frac{V_x}{V}$$