Power:

rate of flow of energy

$$P = \frac{\text{flow of energy}}{\text{second}}$$

units: $\frac{\text{Joule}}{\text{second}}$ = Watt (W)

Joule (J) is unit of energy (or work)

$$1\,J = 1\,kg\,\frac{m^2}{s^2}$$

Impulse & Momentum

Impulse

$$\vec{J} = \int \vec{F}\, dt$$

if $\vec{F}$ is constant,

$$\vec{J} = \vec{F}\,\Delta t$$

In general

$$\vec{J} = \vec{F}_{avg}\,\Delta t$$

$$\vec{F}_{avg} = \frac{1}{\Delta t}\int F\, dt$$

$$\vec{J}_N = (10N \rightarrow)(5s) = 50\,Ns \rightarrow$$

$$\vec{J}_k = (12N \leftarrow)(5s) = 60\,Ns \leftarrow$$

$$\vec{J}_{mg} = (15N \downarrow)(5s) = 75\,Ns \downarrow$$

$$\vec{J}_{N_t} = (15N \uparrow)(5s) = 75\,Ns \uparrow$$

$$\vec{J}_{tot} = 50 \rightarrow + 60 \leftarrow + \cancel{75\uparrow} + \cancel{75\downarrow}$$

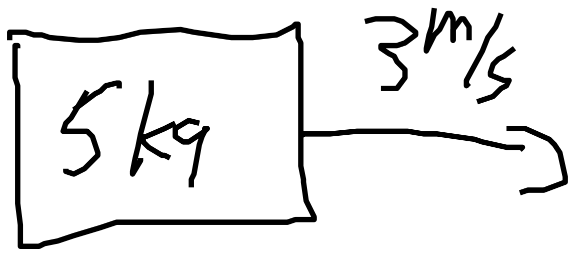
$$= 10\,Ns \leftarrow$$

Momentum

vector

$$\vec{p} = m\vec{v}$$

mass × velocity



$$\vec{p} = (5\,kg)(3\,m/s \rightarrow)$$

$$= 15\,\frac{kg\,m}{s} \rightarrow \qquad 1N = 1\,\frac{kg\,m}{s^2}$$

$$= 15\,Ns \rightarrow$$

Impulse & momentum have the same units

$$\vec{F}_{net} = m\vec{a}$$

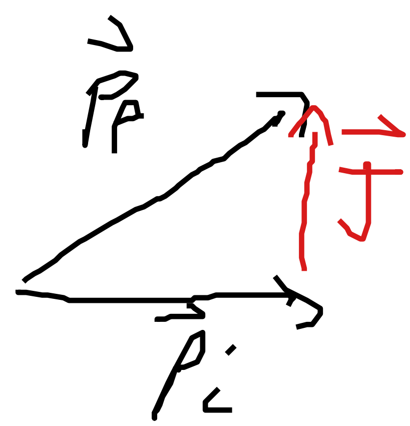
$$\int \vec{F}_{net}\,dt = \int m \frac{d\vec{v}}{dt}\,dt$$

$$\vec{J}_{net} = m\int_{t_i}^{t_f} \frac{d\vec{v}}{dt}\,dt$$

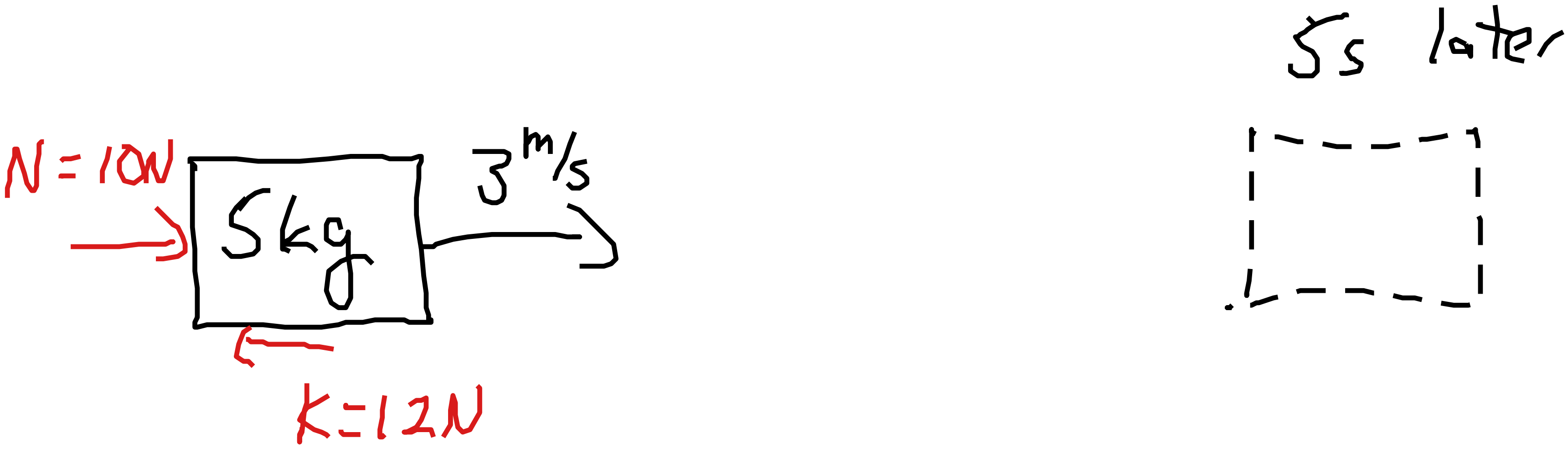
$$\vec{J}_{net} = m\left(\vec{v}_f - \vec{v}_i\right)$$

$$= \vec{p}_f - \vec{p}_i$$

$$\boxed{\vec{J}_{net} = \Delta\vec{p}}$$



$$\vec{p}_f = \vec{p}_i + \vec{J}_{net}$$



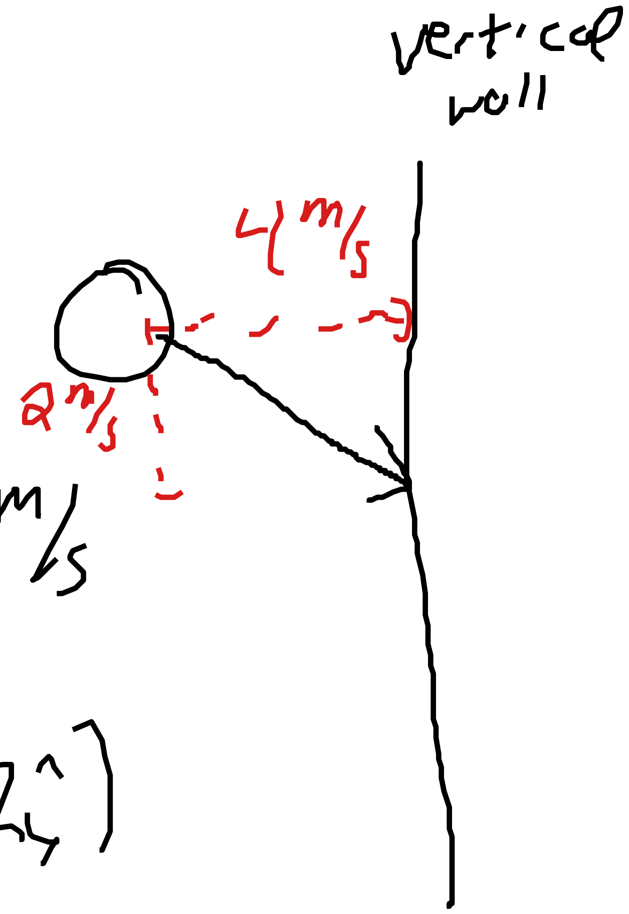
$$\vec{p}_i = (5\,kg)(3\,m/s \rightarrow) = 15\,Ns \rightarrow$$

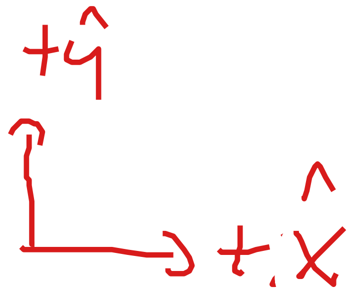
$$\vec{J}_{net} = 10\,Ns \leftarrow$$

$$\vec{p}_f = 15\,Ns \rightarrow + 10\,Ns \leftarrow$$

$$= 5\,Ns \rightarrow$$

$$\vec{v}_f = \frac{\vec{p}_f}{m} = \frac{5\,Ns \rightarrow}{5\,kg} = 1\,m/s \rightarrow$$





$m = 3\,kg$

$$\vec{v}_i = (4\hat{x} - 2\hat{y})\ m/s$$

$$\vec{p}_i = 3(4\hat{x} - 2\hat{y})$$
$$= (12\hat{x} - 6\hat{y})\ Ns$$

$\vec{J} =$ area under curve

$$\vec{J} = \vec{F}_{avg}\Delta t$$

point in same direction

$$\left.\begin{aligned}\vec{F}_{avg} &= F_{avg}(-\hat{x})\\ \vec{J} &= J(-\hat{x})\end{aligned}\right\}\ \boxed{J = F_{avg}\Delta t}$$

$$\vec{p}_f = (12\hat{x} - 6\hat{y}) + J(-\hat{x})$$
$$= (12 - J)\hat{x} - 6\hat{y}\ Ns$$

Momentum & Impulse are particularly useful in collisions

"initial" = "just before collision"

"final" = "just after collision"

e.g. Suppose in previous problem

$$\vec{p}_f = -6\hat{x} - 6\hat{y}$$

$$\Delta t = 8\,ms$$

$$12 - J = -6 \Rightarrow J = 18\,Ns$$

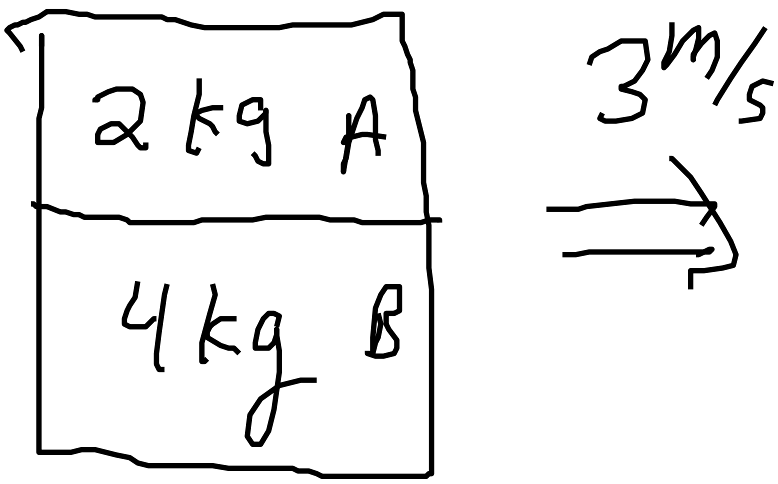
$$F_{avg} = \frac{J}{\Delta t} = \frac{18\,Ns}{8\times10^{-3}\,s} = 2250\,N$$

Compare weight: $mg = (3kg)(9.8) = 29.4\,N$

The force of gravity is <u>much</u> smaller than the total force.

In general, during a collision, only the collision forces matter.

Systems



$$\vec{P}_A = m_A \vec{V}_A = (2\,kg)(3\,m/s \rightarrow)$$
$$= 6\,Ns \rightarrow$$

$$\vec{P}_B = (4\,kg)(3\,m/s \rightarrow) = 12\,Ns \rightarrow$$

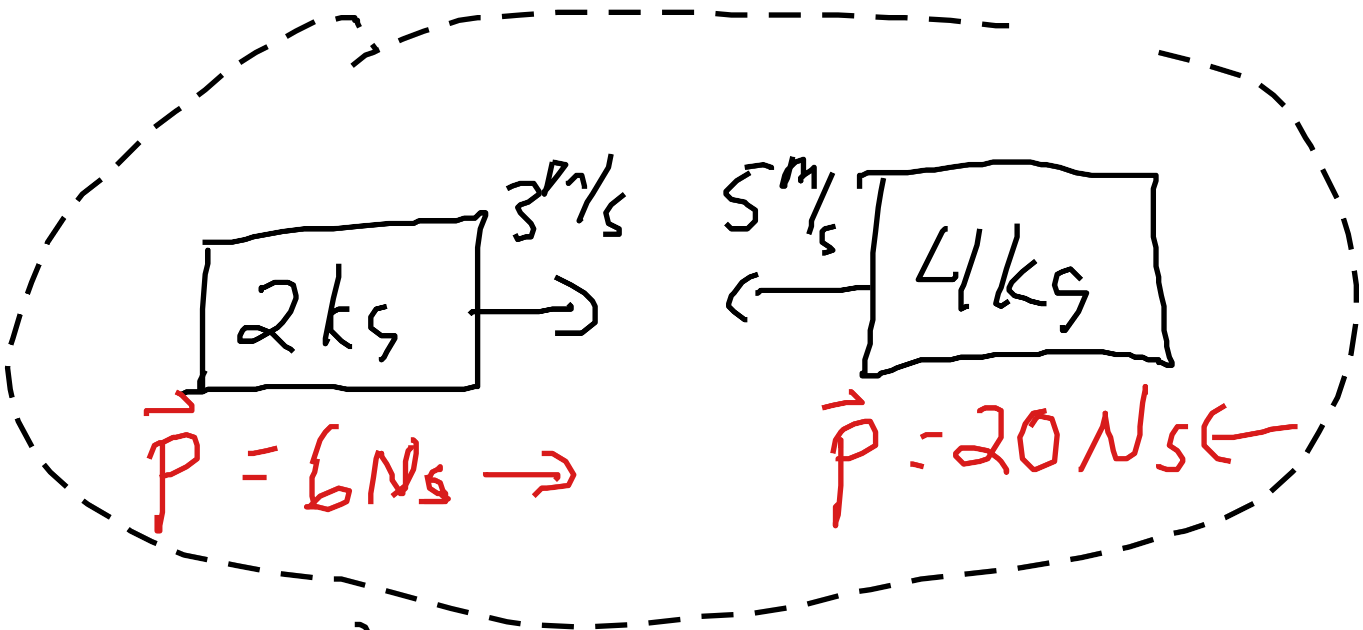
Treat both boxes as a single object

$$\vec{P}_{tot} = (6\,kg)(3\,m/s \rightarrow)$$
$$= 18\,Ns \longrightarrow$$

$$\vec{P}_{tot} = \vec{P}_A + \vec{P}_B$$

Momentum is additive.

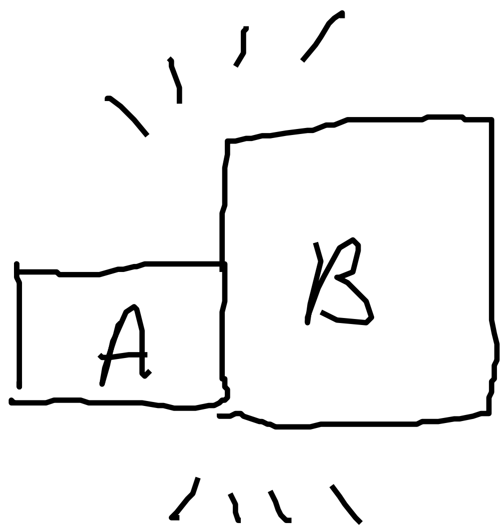
... even if objects move independently.



$$\vec{P}_{tot} = 6 - 20 = -14\,Ns$$

$$= 14\,Ns \leftarrow$$

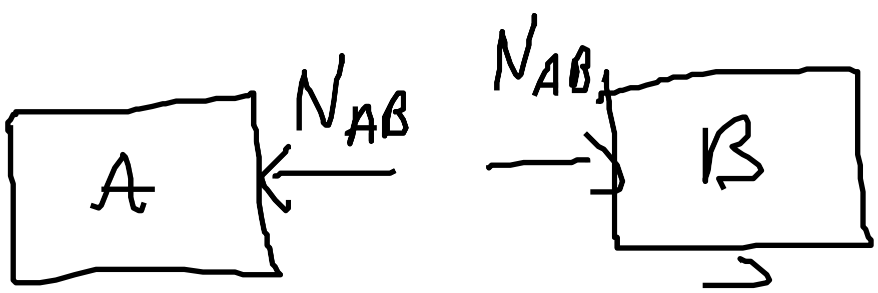
treat both boxes as a "system"



When they collides

only collision forces matter

$$\vec{J}_{tot} = \vec{J}_A + \vec{J}_B$$



$$\vec{J}_A = N_{AB} \leftarrow \qquad \vec{J}_B = N_{AB} \rightarrow$$

$$\vec{J}_{tot} = N_{AB} \leftarrow + N_{AB} \rightarrow = 0.$$

We say that $\vec{N}_{AB}$ & $\overleftarrow{N}_{AB}$ are "internal forces" (internal to the system)

When calculating total impulse (or total force) on the system, they cancel.

$$\vec{P}_{f,tot} = \vec{P}_{i,tot} + \vec{J}_{\substack{tot \\ external}}$$

for a fast collision,

$$\vec{J}_{\substack{tot \\ external}} \simeq \vec{0}$$

In a fast collision,

$$\vec{P}_{f,tot} = \vec{P}_{i,tot}$$

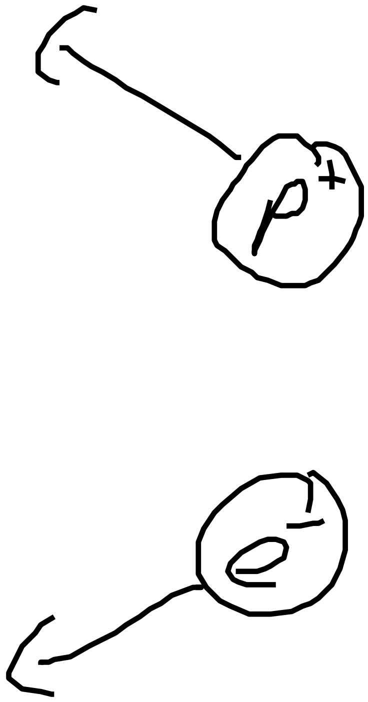
Conservation of Momentum

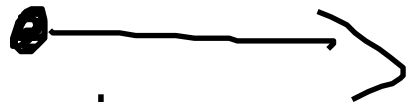
(this equation requires that we include all of interacting parts in our system.)

$\vec{P}_i = 0$ neutron $v = 0$

$\vec{P}_f = m_p \nwarrow + m_e \swarrow \neq 0$

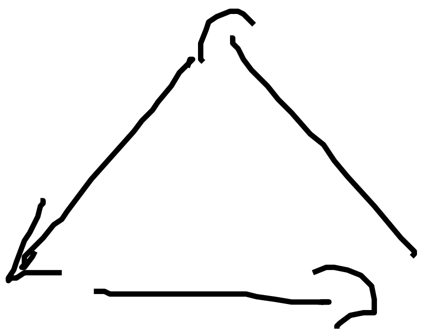
can't add to P_i





little neutral one -

<u>neutrino</u>



now it can

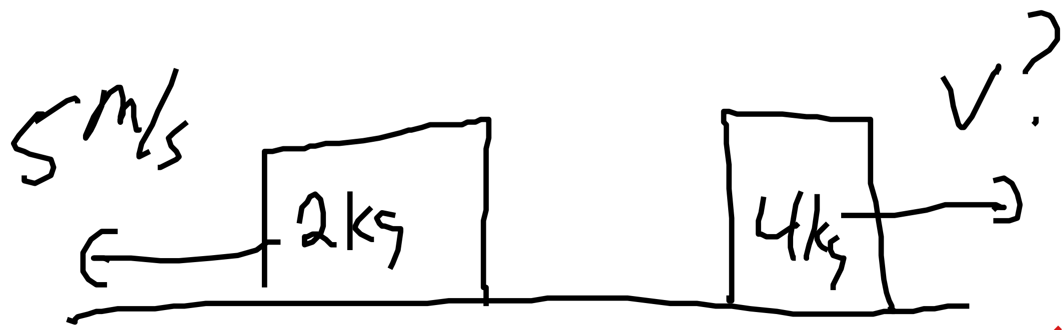
$\rightarrow +$ before

$\vec{P}_{iA} = 6\,Ns \rightarrow$ $\vec{P}_{iB} = 20\,Ns \leftarrow$

$= +6\,Ns$ $= -20\,Ns$

$P_{i\,tot} = -14\,Ns$

after



$P_{fA} = -10\,Ns$ $P_{fB} = (4)v$

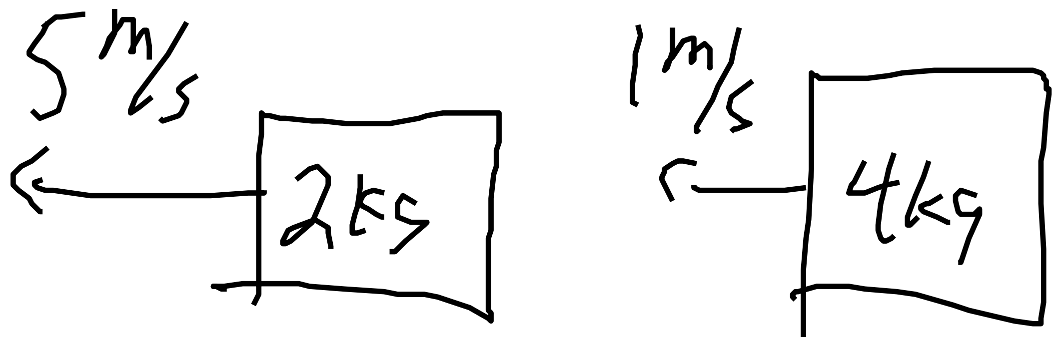
$P_{f,tot} = -10 + 4v$

$P_f = P_i$

$-10 + 4v = -14$

$4v = -4 \rightarrow v = -1\,m/s$

$v = -1\ m/s$ means that block B moves to the left.



Only immediately after the collision

To determine both velocities we will need more information (usually related to energy).

initial

$$E_{k,A,i} = \frac{1}{2}mv^2 = \frac{1}{2}(2)(3)^2 = 9\,J$$

$$E_{kBi} = \frac{1}{2}mv^2 = \frac{1}{2}(4)(5)^2 = 50\,J$$

$$E_{ki} = 59\,J$$

final

$$E_{k,A,f} = \frac{1}{2}(2)(5)^2 = 25\,J$$

$$E_{kBf} = \frac{1}{2}(4)(1)^2 = 2\,J$$

$$E_{kf} = 27\,J$$

$$\Delta E_k = -32\,J$$

sound, heat, ...

Elastic collision:

- momentum and energy conserved
- common at subatomic level

almost impossible at our scale

Inelastic collision: energy is lost

Superelastic collisions: energy is gained e.g. explosion

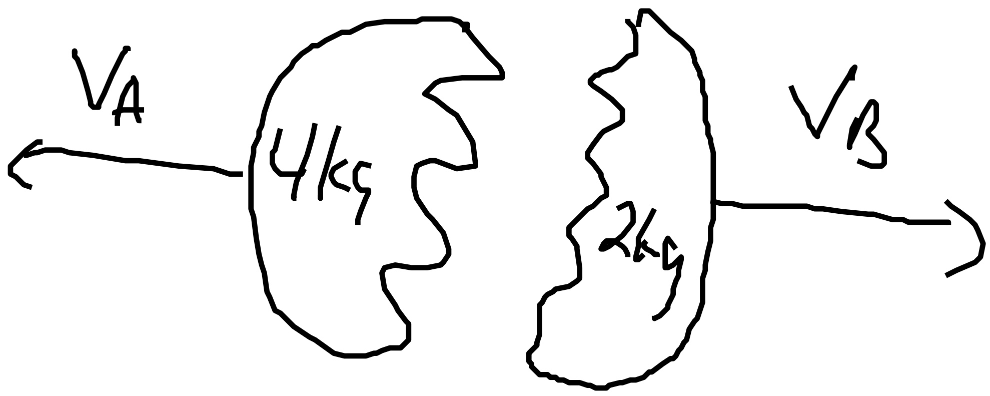
$\vec{P}_i = 0$

$V = 0 \quad E_k = 0$

explodes!

$\rightarrow +$

$\Delta E_k = 100\,J$



$\vec{P}_{fA} = 4(-V_A) \quad \vec{P}_{fB} = 2\,V_B$

$E_{kfA} = \frac{1}{2}(4)V_A^2 \quad E_{kfB} = \frac{1}{2}(2)V_B^2$

$2V_A^2$

V_B^2

$$P_{f\,tot} = P_{i,tot}$$

$$-4v_A + 2v_B = 0$$

$$E_f = E_i + \Delta E$$

$$2v_A^2 + v_B^2 = 0 + 100$$

$$4v_A = 2v_B \Rightarrow v_B = 2v_A$$

v_B

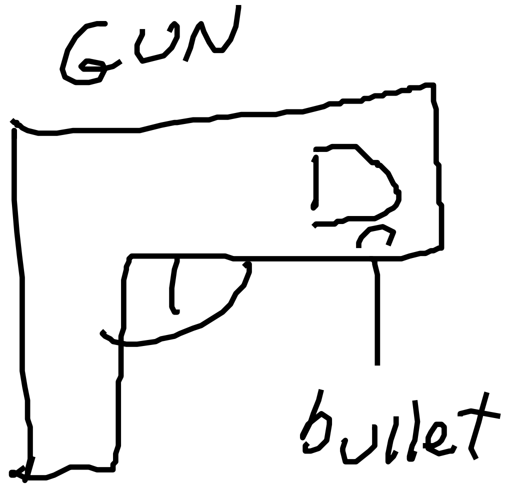
$$2v_A^2 + (2v_A)^2 = 100$$

$$2v_A^2 + 4v_A^2 = 100$$

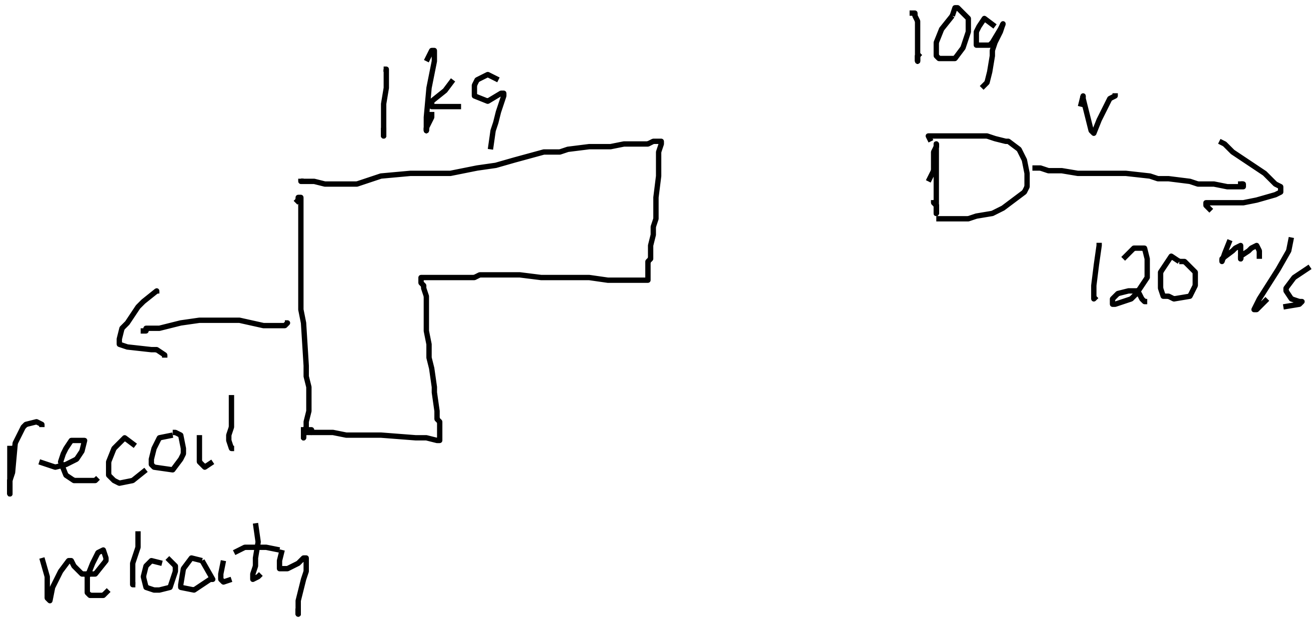
$$6v_A^2 = 100 \rightarrow v_A = \sqrt{\frac{100}{6}}$$

$$= 4.1\,\text{m/s}$$

$$v_B = 2v_A = 8.2\,\text{m/s}$$



$$P_i = 0$$



$$(1\,kg)(-V_r) + (0.01\,kg)(120) = 0$$

$$V_r = \frac{1.2}{1\,kg} = 1.2\ m/s$$