1 These are notes from a Q&A session, and are rather disjoint.

A $m = 500\,kg$ car moving $30\ m/s$ collides with a pillar. The collision takes $0.4\,s$. How much average power is expended?

$$\text{Power} = \frac{\text{Energy loss}}{\text{time}}$$

$$\text{Energy }\underline{\text{loss}} = |E_f - E_i|$$

$$E_f = 0$$

$$E_i = \frac{1}{2} m v^2$$

$$E_i = \frac{1}{2}(500)(30)^2$$

$$= (250)(900)$$

$$= 225{,}000\ \text{J}$$

$$E_f = 0$$

Energy lost is 225,000 J

$$\text{Power} = \frac{225{,}000\ \text{J}}{0.4\ \text{s}}$$

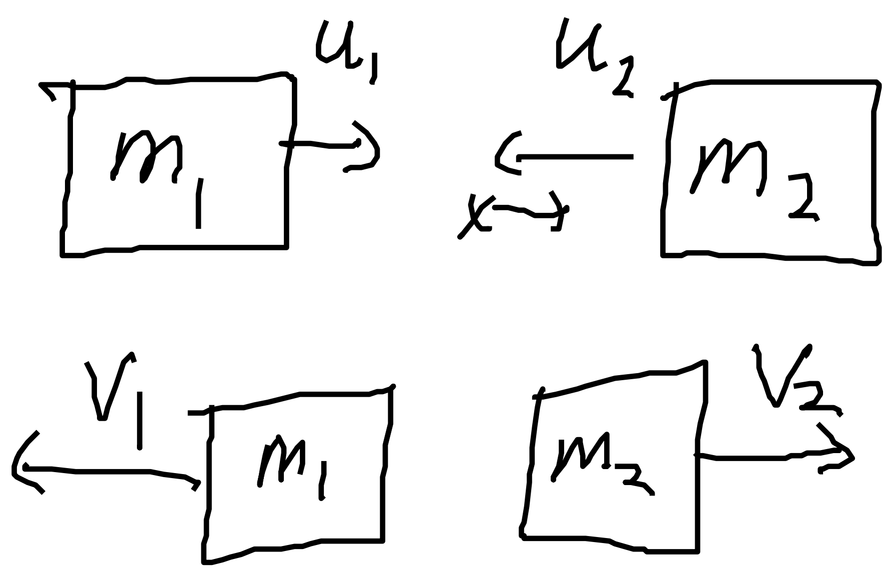
$$= 562{,}500\ \text{W}$$

$$= 562.5\ \text{kW}$$

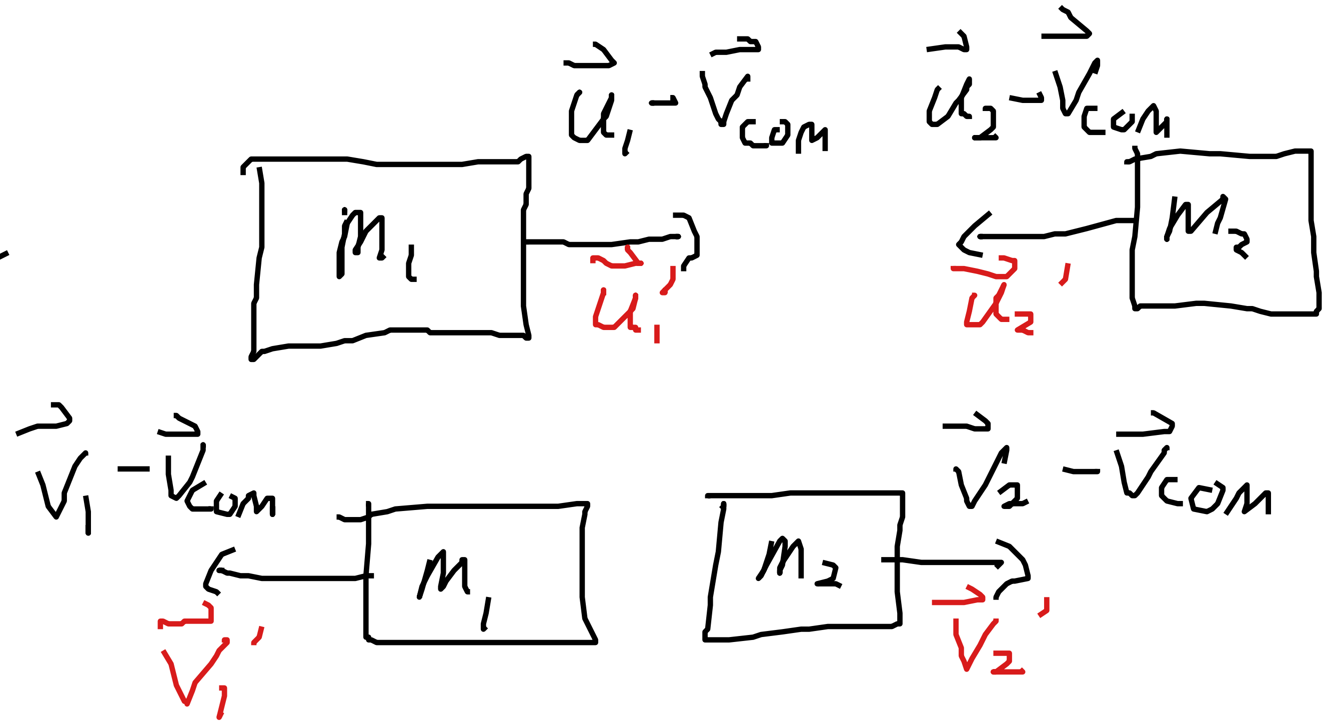
$$X_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$V_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{P_{tot}}{M_{tot}}$$

Lab Frame



COM Frame



In COM frame,

$$m_1 v_1' + m_2 v_2' = p' = 0$$

$$\vec{v}_2' = -\frac{m_1}{m_2}\vec{v}_1'$$

$$\vec{u}_2' = -\frac{m_1}{m_2}\vec{u}_1'$$

$$E_{ki}' = \frac{1}{2} m_1 |\vec{u}_1'|^2 + \frac{1}{2} m_2 |\vec{u}_2'|^2$$

$$E_{kf}' = \frac{1}{2} m_1 |\vec{v}_1'|^2 + \frac{1}{2} m_2 |\vec{v}_2'|^2$$

$$E_{ki}' = \frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 \left(-\frac{m_1}{m_2} u_1'\right)^2$$

$$\vec{u}_1' = \vec{u}_1 - \vec{V}_{com}$$

$$= \vec{u}_1 - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2}$$

$$= \frac{\vec{u}_1(m_1 + m_2)}{m_1 + m_2} - \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2}$$

$$= \frac{\cancel{m_1\vec{u}_1} + m_2\vec{u}_1 - \cancel{m_1\vec{u}_1} - m_2\vec{u}_2}{m_1 + m_2}$$

$$\vec{u}_1' = \frac{m_2}{m_1 + m_2}\left(\vec{u}_1 - \vec{u}_2\right)$$

$$\vec{u}_2' = \frac{m_1}{m_1 + m_2}\left(\vec{u}_2 - \vec{u}_1\right)$$

$$E'_{k_i} = \frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2$$

$$= \frac{1}{2} m_1 \left(\frac{m_2}{m_1+m_2} (u_1 - u_2) \right)^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_1+m_2} (u_2 - u_1) \right)^2$$

$$= \frac{1}{2} \frac{m_1 m_2^2}{(m_1+m_2)^2} (u_1 - u_2)^2 + \frac{1}{2} \frac{m_2 m_1^2}{(m_1+m_2)^2} (u_2 - u_1)^2$$

$$= \frac{1}{2} \frac{m_1 m_2}{(m_1+m_2)^{\cancel{2}}} (u_1 - u_2)^2 \cancel{(m_2 + m_1)}$$

$u_1' - u_2'$

$$E'_{k_i} = \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} (u_1 - u_2)^2$$

$$E'_{kf} = \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} (v_1 - v_2)^2$$

$v_1' - v_2'$

Relative velocities of blocks 1 & 2 remain same even if you shift to com

If elastic collision
& energy is conserved,

$$E_{ki}' = E_{kf}'$$

$$(u_1' - u_2')^2 = (v_1' - v_2')^2$$

$$u_2' = -\frac{m_1}{m_2} u_1'$$

$$u_1' - u_2' = u_1' + \frac{m_1}{m_2} u_1'$$

$$= u_1' \left(1 + \frac{m_1}{m_2}\right)$$

$$\cancel{E_{ki}'} \cancel{=} \cancel{\frac{1}{2}} \cancel{\frac{m_1 m_2}{m_1+m_2}} \cancel{u_1'^2} \cancel{\left(\frac{m_1+m_2}{m_2}\right)^2}$$

$$v_1' - v_2' = v_1'\left(1 + \frac{m_1}{m_2}\right)$$

$$E_{ki}' = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1' - u_2')^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1')^2\left(1 + \frac{m_1}{m_2}\right)^2$$

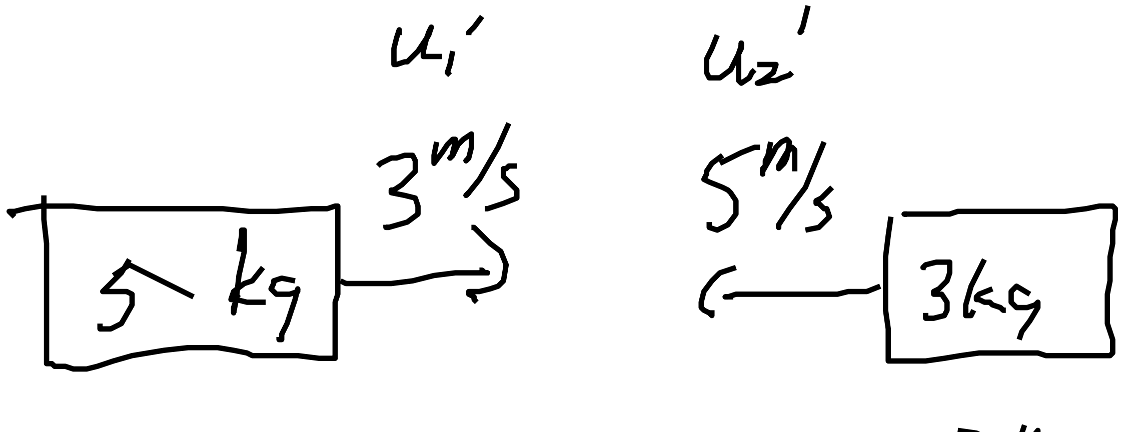
$$E_{kf}' = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(v_1')^2\left(1 + \frac{m_1}{m_2}\right)^2$$

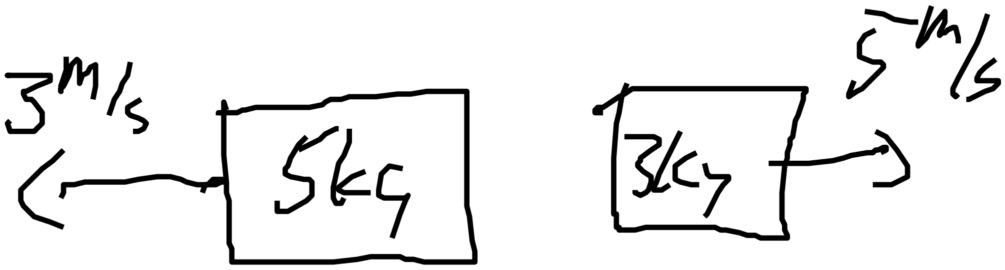
if $E_{ki}' = E_{kf}'$

$$u_1'^2 = v_1'^2 \rightarrow v_1' = \pm u_1'$$

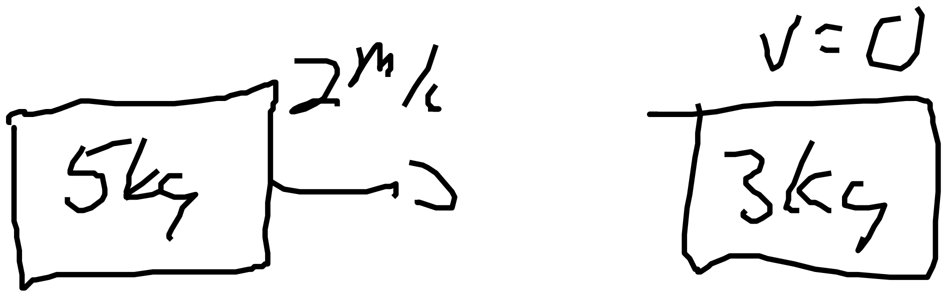
$$u_2'^2 = v_2'^2 \rightarrow v_2' = \pm u_2'$$

In COM frame,





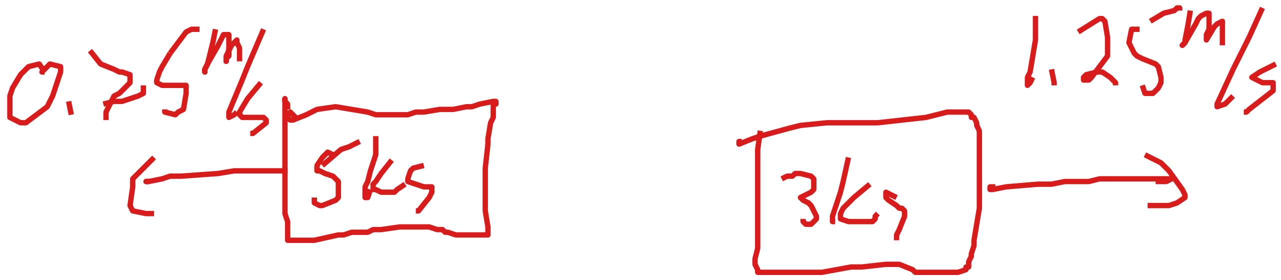
e.g
elastic



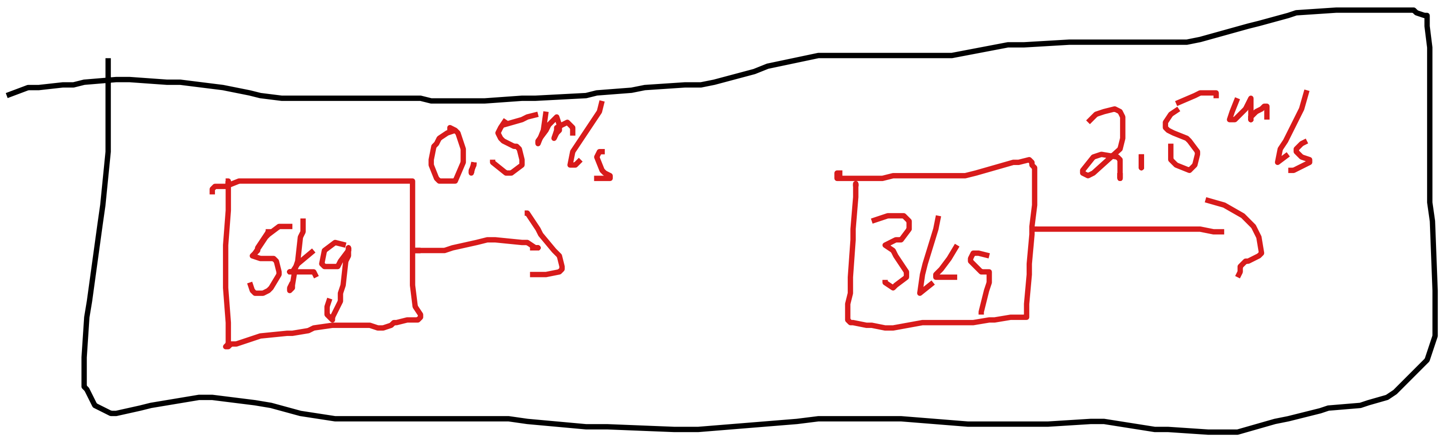
1) Switch to com

- find $P_{tot} = +10 + 0 = 10$
- find $V_{com} = \frac{P_{tot}}{M} = \frac{10}{8} = 1.25\,m/s$
- subtract V_{com} from all velocities

2) Flip the velocities



3) Add back V_{com} to switch to lab frame



Partially inelastic collision

$$e = \frac{|\vec{V}_1 - \vec{V}_2|}{|\vec{u}_1 - \vec{u}_2|} = \frac{|\vec{V}_1' - \vec{V}_2'|}{|\vec{u}_1' - \vec{u}_2'|}$$

coefficient of restitution

$e = 1$: elastic collision

$e = 0$: maximally inelastic collision

$V_1 = V_2$

$e > 1$: superelastic collision

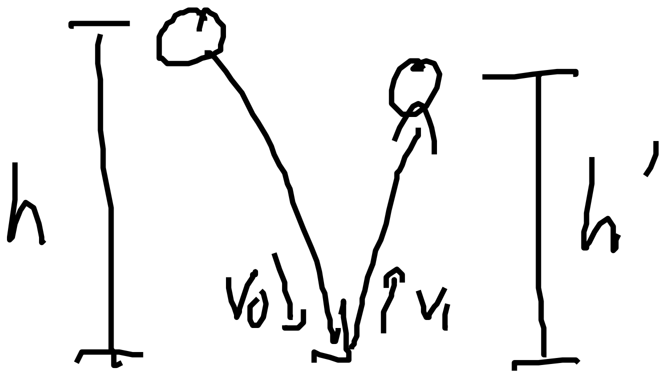
$0 < e < 1$: partially inelastic collision

$$\frac{KE_f' = \cancel{\text{mass stuff}}\ |v_1 - v_2|^2}{KE_i' = \cancel{\text{mass stuff}}\ |u_1 - u_2|^2}$$

$$\frac{KE_f'}{KE_i'} = \frac{|v_1 - v_2|^2}{|u_1 - u_2|^2} = e^2$$

e^2: fraction of energy retained during collision

$1 - e^2$: fraction of energy lost



$$mgh = \frac{1}{2} m v_0^2 = KE_i$$

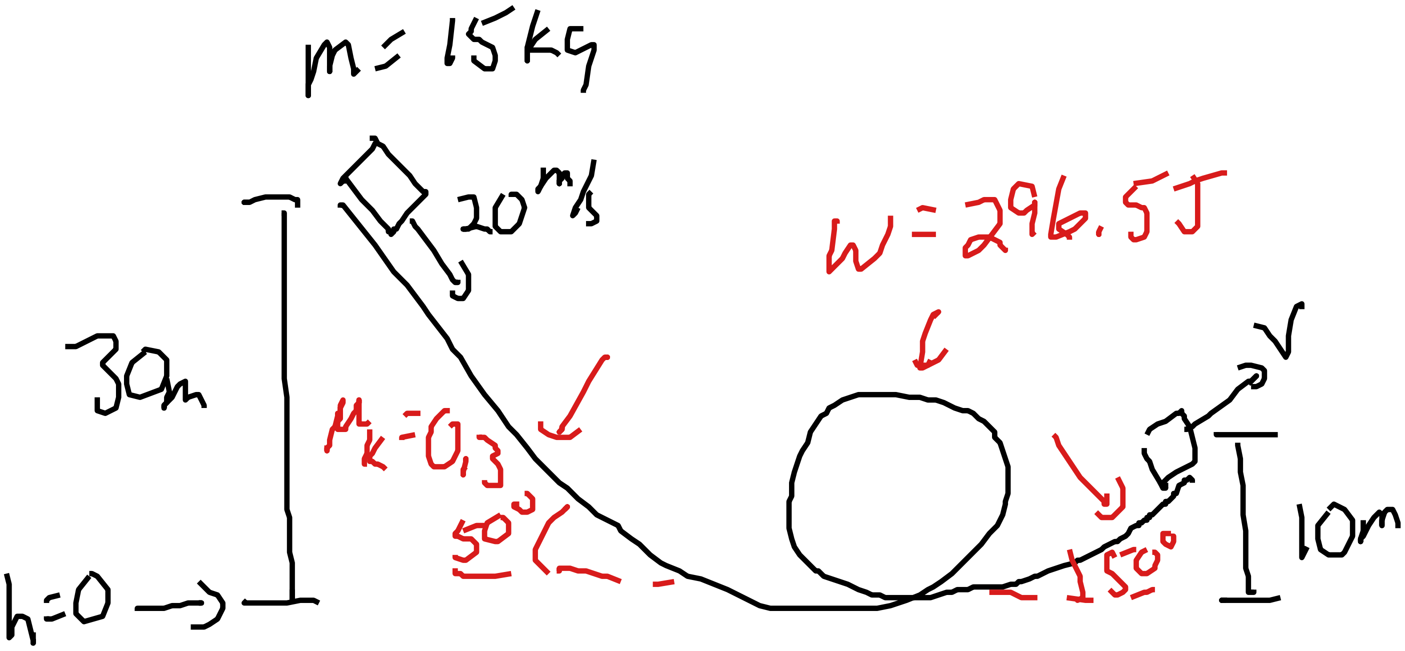
$$mgh' = \frac{1}{2} m v_1^2 = KE_f$$

$$e^2 = \frac{KE_f}{KE_i} = \frac{\cancel{mg}h'}{\cancel{mg}h}$$

$$e = \sqrt{\frac{h'}{h}}$$

$$E_f = E_i + \Delta E$$

$$E_f = E_i + W$$



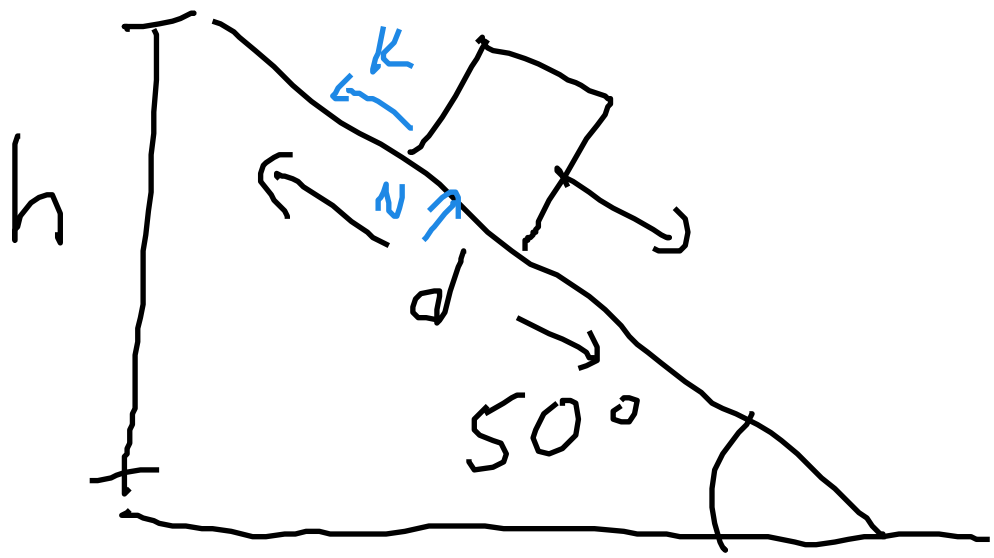
$$E_i = \frac{1}{2}(15)(20)^2 + (15)(9.8)(30)$$

$$E_f = \frac{1}{2}mv^2 + mgh$$

$$= \frac{1}{2}(15)v^2 + (15)9.8(10)$$

$$E_f = E_i - W_{\text{on loop}} - W_{\text{on 1st slope}} - W_{\text{on 2nd slope}}$$

Energy lost on a slope



$$W = -Kd$$

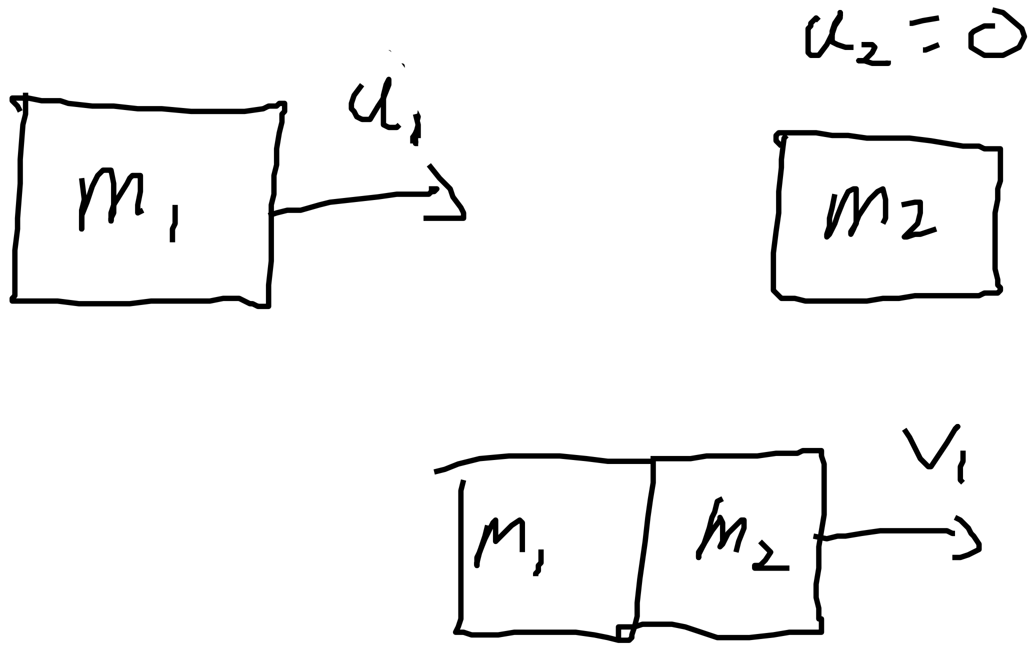
$$= -\mu_k N d$$

$$= -\mu_k mg(\cos 50^\circ) d$$

$$\frac{h}{d} = \sin 50^\circ \rightarrow d = \frac{h}{\sin 50^\circ}$$

$$W = -\mu_k mg \frac{\cos\theta}{\sin\theta} h$$

Maximally Inelastic Collision



$$P_i = m_1 u_1$$
$$P_f = (m_1 + m_2) V_1$$

equal because collision

$$m_1 u_1 = (m_1 + m_2) V_1$$

$$\frac{m_1}{m_1 + m_2} u_1 = V_1$$