Fluids – liquids & gases

- Density ρ "rho" $\frac{kg}{m^3}$
- Pressure P $\frac{Force}{Area}$

units: Pascals $1\,Pa = 1\,\frac{N}{m^2}$

Fluids apply a pressure to every surface they touch

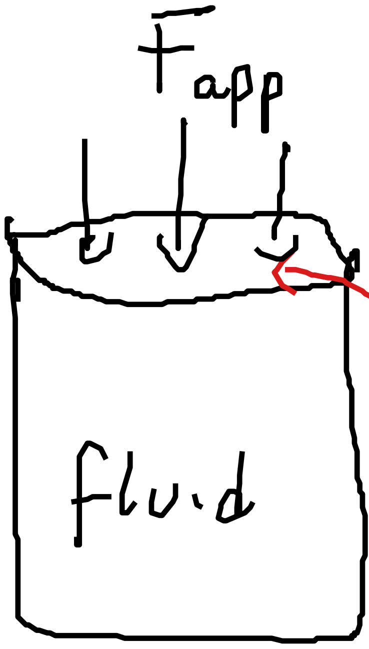
Atmospheric pressure

$P_{atm} = 1.013 \times 10^5\,Pa$

$= 101\,kPa$

$=$ "1 atm"

$P = 0$ in vacuum



piston has area A

By applying a force to the piston, we add to the pressure on the fluid.

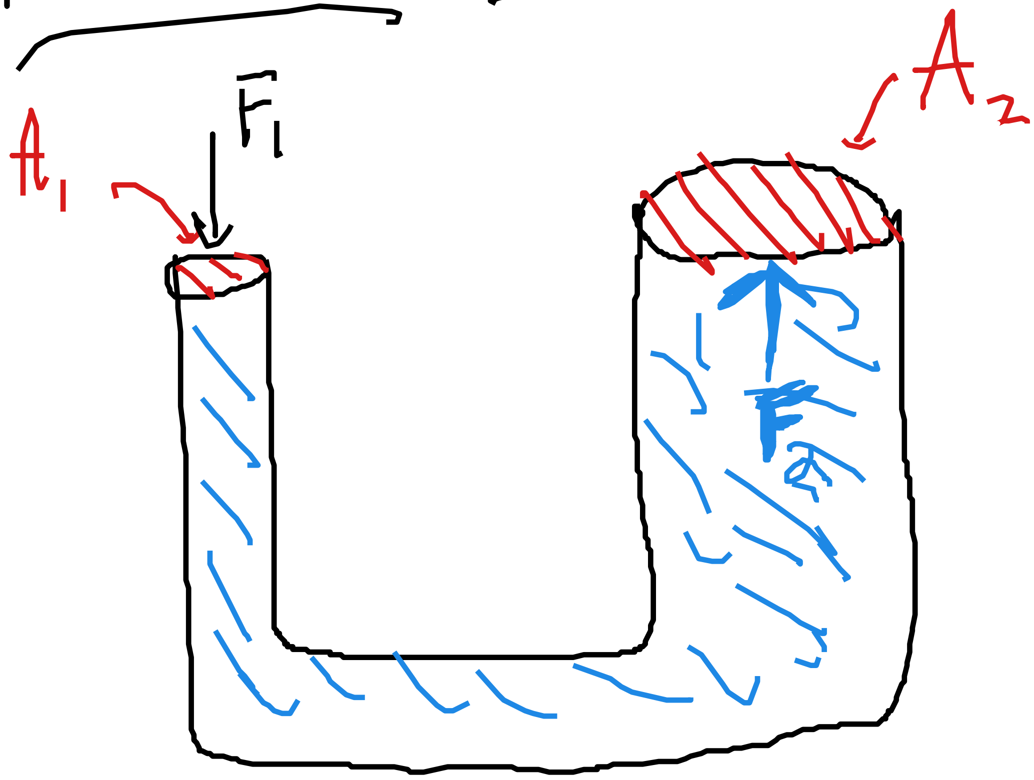
$$P_{\text{on fluid}} = \underbrace{P_{atm} + \frac{F_{app}}{A_{piston}}}$$

N3L says

Pressure at surface of fluid is same

The pressure throughout the fluid increases by F/A, if fluid is incompressible (liquid).

Pascal's Principle



filled with fluid with density ρ_f

- Push down on smaller piston with force F_1
- Increases pressure in fluid everywhere by F_1/A_1.
- Pressure beneath second piston increases by F_1/A_1

- There is an additional force of $F_2 = PA$

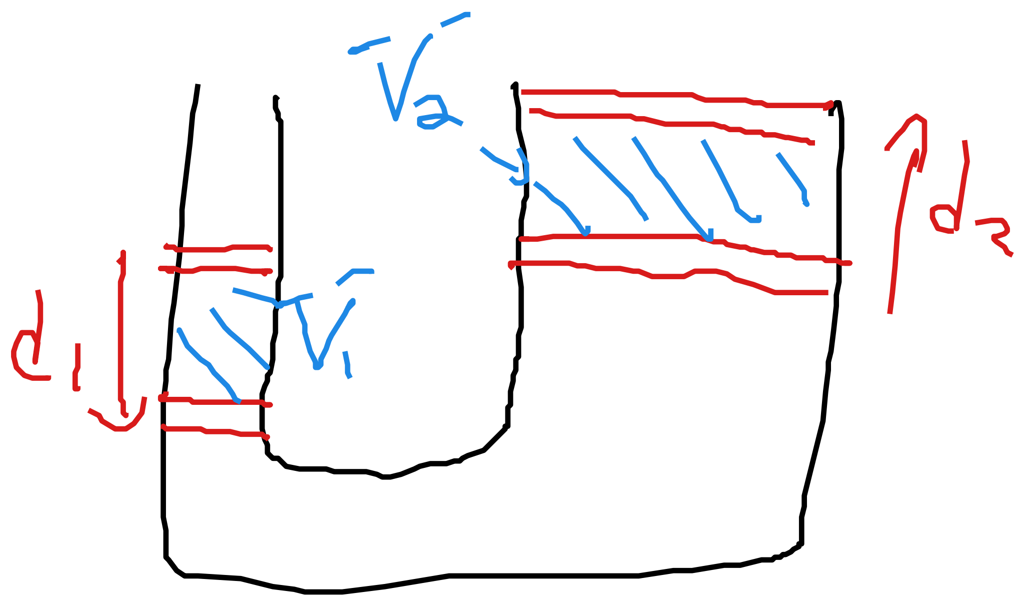
$$F_2 = \left(\frac{F_1}{A_1}\right) A_2$$

$$F_2 = F_1 \frac{A_2}{A_1}$$

This is a hydraulic jack

it amplifies an applied force.

To raise the second piston by distance d_2 we need to push first piston down by d_1



$$\vec{V_1} = \vec{V_2}$$

$$A_1 d_1 = A_2 d_2$$

$$d_2 = \frac{A_1}{A_2} d_1$$

Work done on piston 1

$$W_1 = F_1 d_1$$

Work done on piston 2

$$W_2 = F_2 d_2$$

$$= \left(F_1 \frac{\cancel{A_2}}{\cancel{A_1}}\right)\left(d_1 \frac{\cancel{A_1}}{\cancel{A_2}}\right)$$

$$W_2 = W_1$$

Buoyancy $\vec{F}$orce

Any object in a fluid feels an upward force

$$\vec{B} = \rho_f \bar{V}_{disp}\, g \uparrow$$

weight of displaced water

- ρ_f is density of fluid
- $\bar{V}_{disp}$ is volume of fluid that had to be moved out of the way to put object there
- $g = 9.8\ N/kg = 9.8\ m/s$

(Archimedes Principle)

For submerged objects,

$$\bar{V}_{disp} = \bar{V}_{object}$$

e.g. humans in air

$$\rho_f = 1.2\ kg/m^3$$

$$\bar{V}_{human} = 0.06\ m^3$$

$$B_{human} = (1.2)(0.06)(9.8)$$

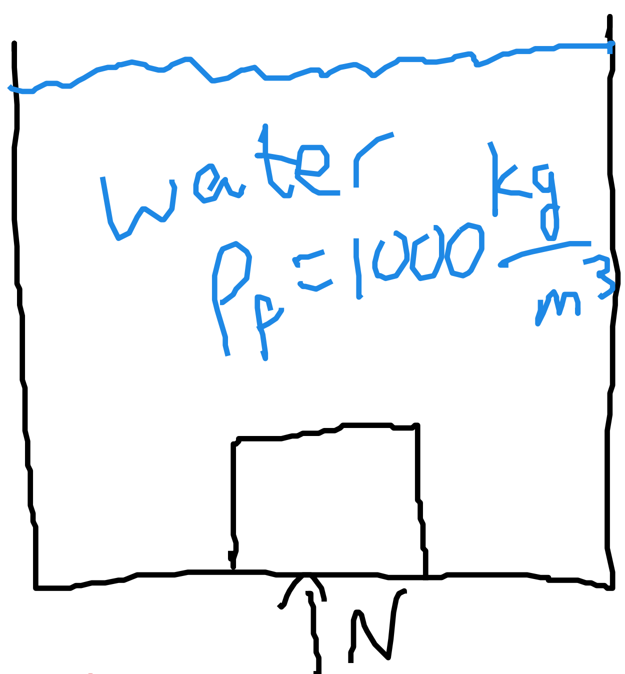
$$= 0.7\ N$$

If $B <$ weight

object will sink

$$\rho_f \cancel{V_{obj}} \cancel{g} < \overbrace{(\rho_{obj} \cancel{V_{obj}})}^{m} \cancel{g}$$

$$\rho_f < \rho_{obj}$$

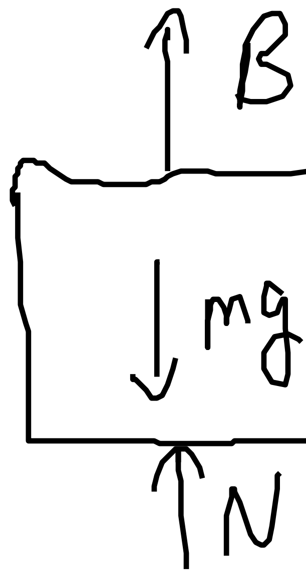


$\rho_{obj} = 3000 \, kg/m^3$

$m = 6 \, kg$

What is N?

$$\rho = \frac{m}{V} \rightarrow V = \frac{m}{\rho} = \frac{6}{3000} = 0.002 \, m^3$$



$$B + N = mg$$

$$\rho_f V_d g + N = mg$$

$$N = (m - \rho_f V_d)g$$

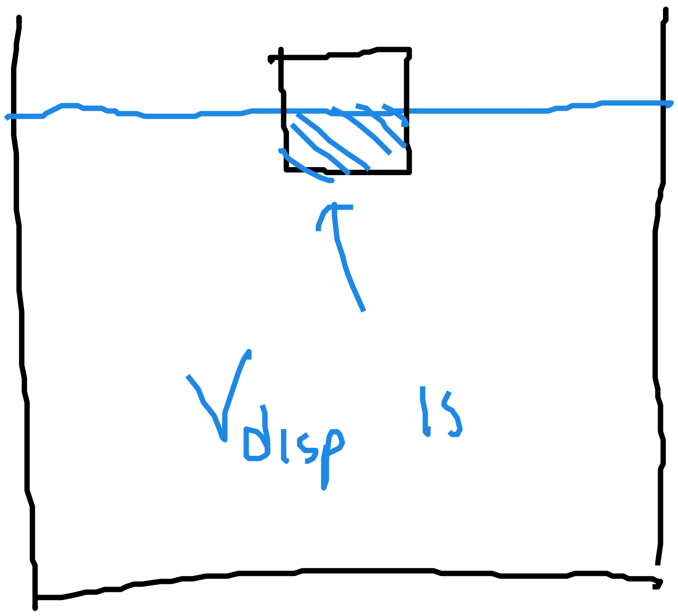
$$= (6 - (1000)V_{obj})(9.8)$$

$$= (6 - (1000)(0.002\,m^3))(9.8)$$

$$= \boxed{39.2\,N}$$

$\rho_{obj} < \rho_f$

float



V_{disp} is V that is under the fluid

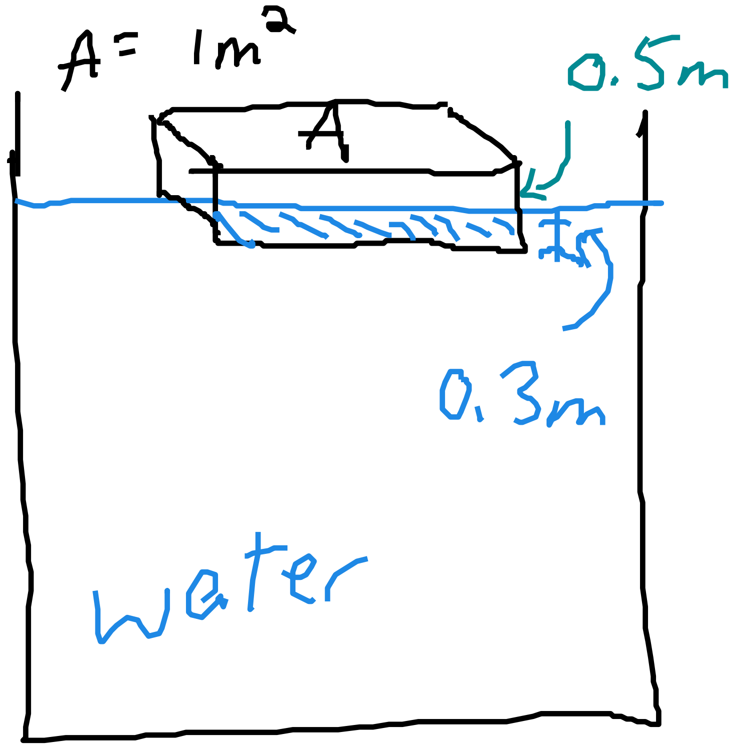
It reaches equilibrium where

$$mg = B$$

$$(\rho_{obj} V_{obj})\cancel{g} = \rho_f V_{disp} \cancel{g}$$

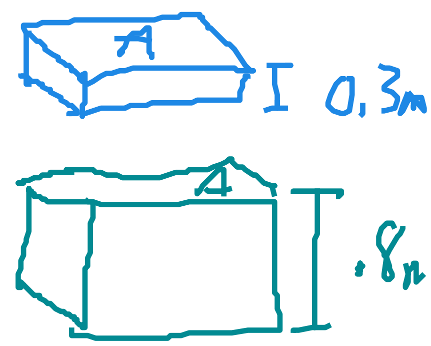
At floating equilibrium

$$\frac{\rho_{obj}}{\rho_f} = \frac{V_{disp}}{V_{obj}}$$



What is density of block?

underwater part



$$\rho_{obj} = \rho_f \frac{V_{disp}}{V_{obj}}$$

$$= \left(1000\frac{kg}{m^3}\right)\frac{(1m^2)(0.3m)}{(1m^2)(0.8m)}$$

$$= (1000)\frac{.3}{.8}$$

fraction of block that is underwater

$$= \frac{3}{8}(1000)$$

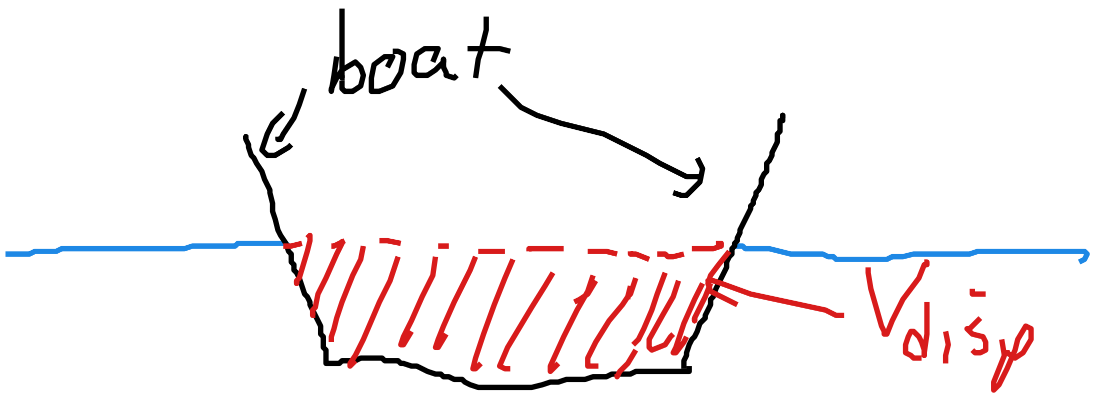
$$= 375\ kg/m^3$$

Ice has a density of $\rho = 917\ kg/m^3$

$(\rho = 0.917\ g/cm^3)$

When ice floats, 91.7% of it is underwater.

How about boats?



We look at average density.

Most of V_{disp} is filled with air with $\rho_{air} << \rho_{water}$

So average density of boat is less than average density of water - floats.

How about hot-air balloons?

- most materials get less dense as they are heated including air

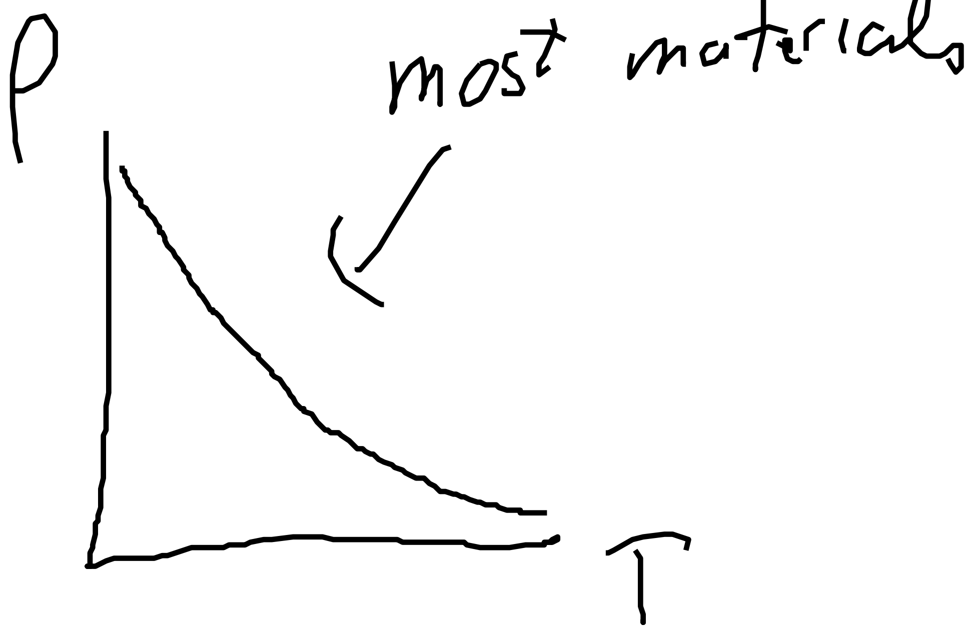
$\rho_{hot\ air} < \rho_{air}$

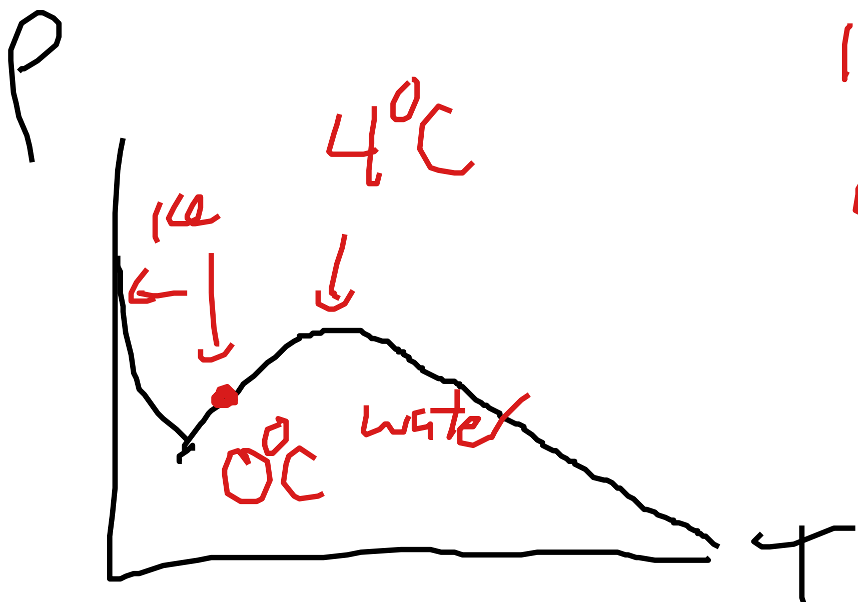
If $\rho_{avg} < \rho_{air}$

then it can float

Aside:

water is weird





ice is less dense than liquid water

Water in Motion

capital phi

- Flow rate or flux

$$\Phi = \frac{\text{Volume}}{\text{time}} = \frac{V}{\Delta t}$$

units: $\frac{m^3}{s}$

e.g. faucet

$$\Phi = \frac{2L}{10s}$$

$1dm = 0.1m$

$$1L = 1\,dm^3$$
$$= (1\,dm)^3 = (0.1m)^3$$
$$= 0.001m^3$$

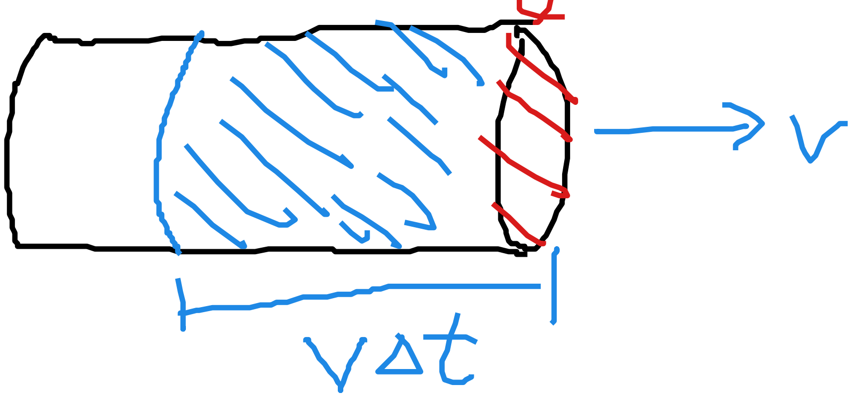
$$1m^3 = 1000L$$

$$\Phi = \frac{0.002\ m^3}{10\ s}$$

$$= 2 \times 10^{-4}\ m^3/s$$

water flowing out of a faucet

cross-sectional area A



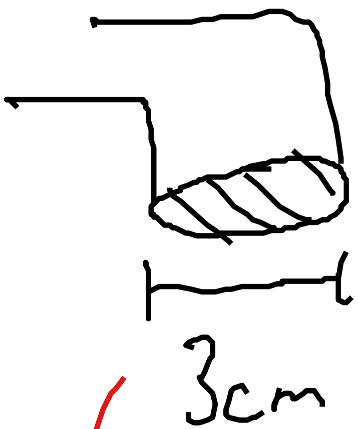
in time Δt,

$$\frac{V}{\Delta t} = \frac{A(v\Delta t)}{\Delta t}$$

$$\boxed{\Phi = \frac{V}{\Delta t} = \underset{m^2}{A}\ \underset{m/s}{v}}$$

Think about our faucet

$$\Phi = 2\times10^{-4}\ m^3/s$$



How fast is water moving?

$$2\times10^{-4} = A\,\underline{v}$$

$$r = 1.5cm = 0.015m$$

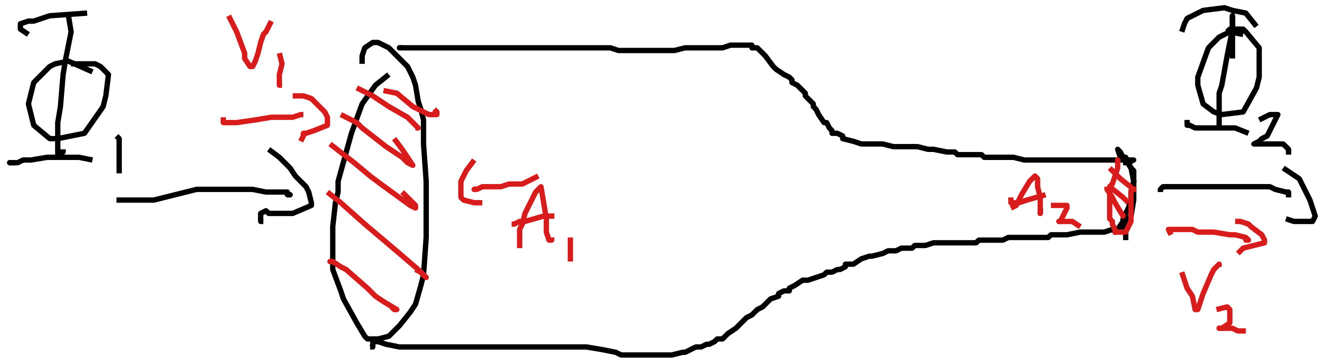
$$A = \pi r^2 = \pi(0.015m)^2$$

$$= 7.1\times10^{-4}\ m^2$$

$$2\times10^{-4} = (7.1\times10^{-4})\,v$$

$$v = \frac{2}{7.1} = 0.28\ m/s.$$

For an incompressible fluid flowing steadily,



$$\Phi_{in} = \Phi_{out}$$

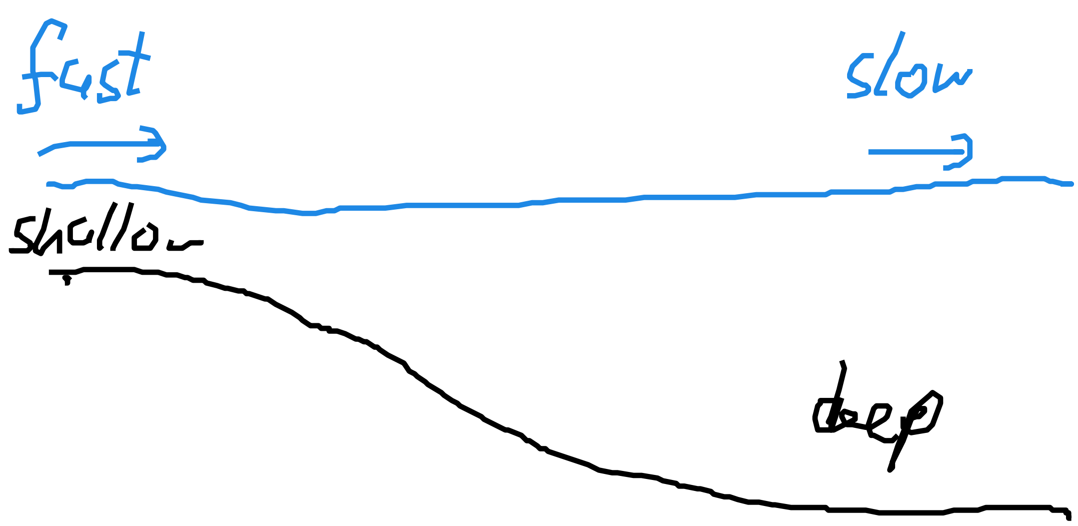
$$A_1 V_1 = A_2 V_2$$

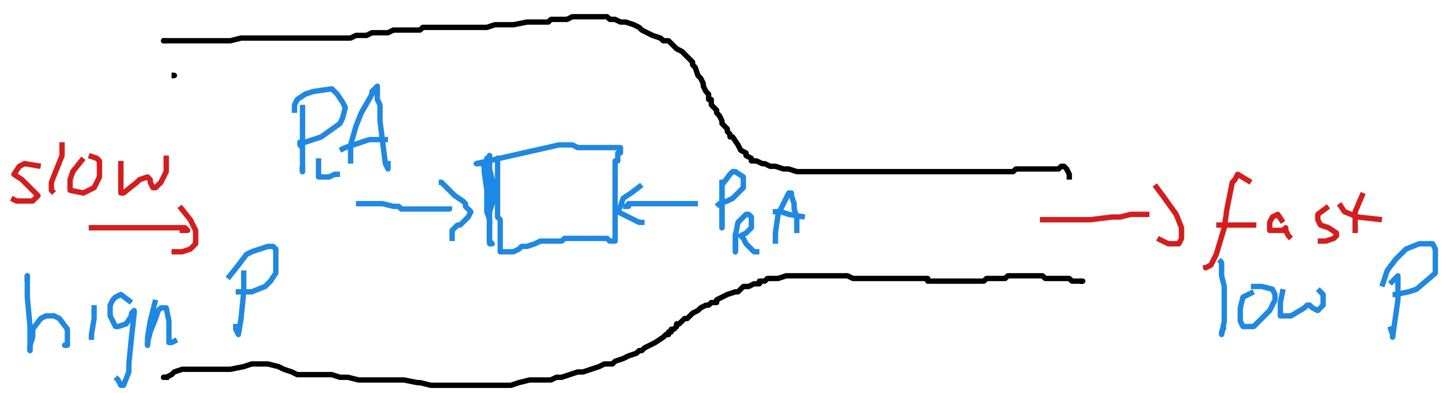
if A_2 is smaller than A_1
V_2 is bigger than V_1

e.g. put your thumb over a hose opening — water gets faster

e.g. "slow waters run deep"

as rivers get deeper, they flow more slowly





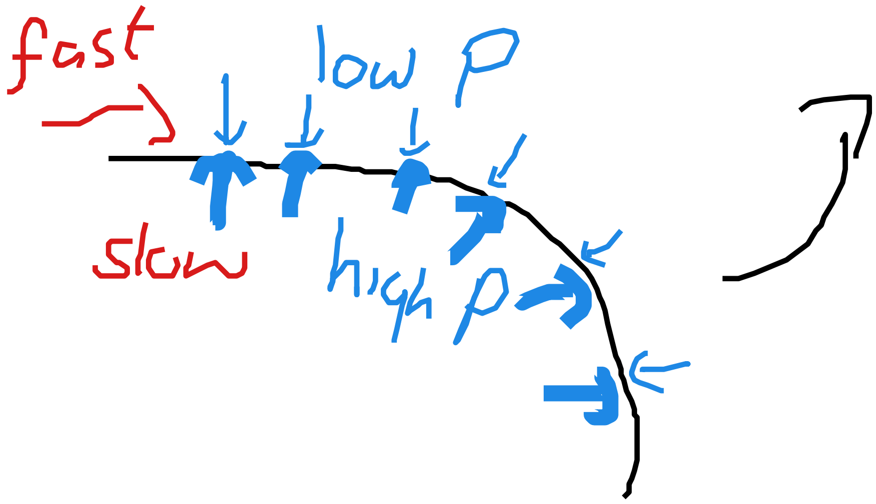
$\longrightarrow$ acceleration

$\longrightarrow$ net force

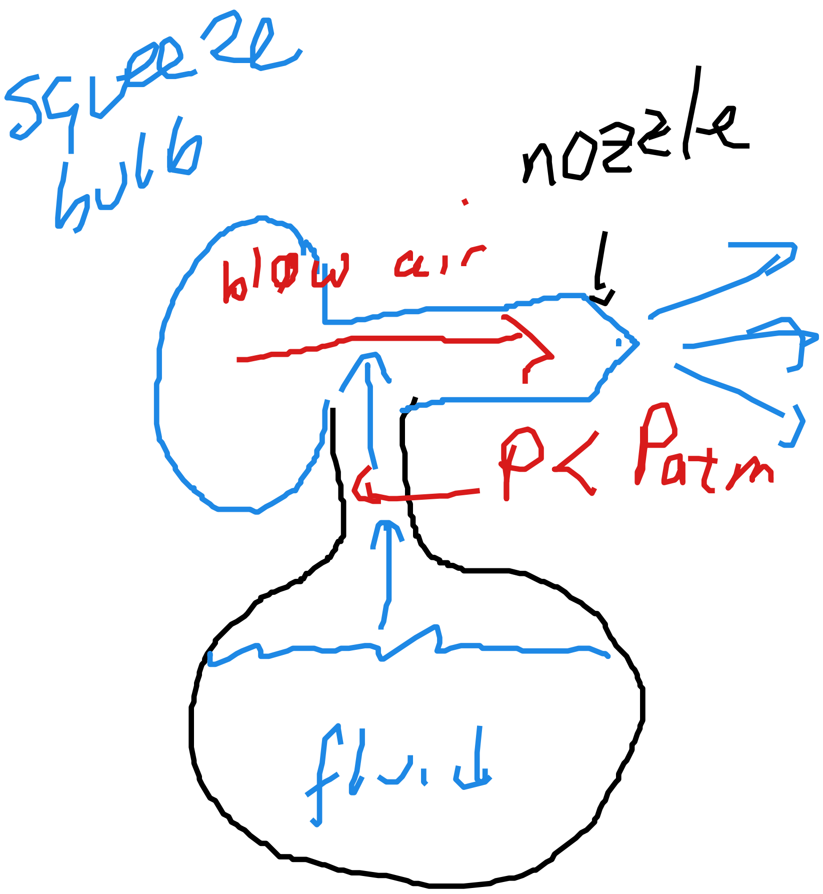
$$P_L > P_R$$

Bernoulli's Principle

- pressure in a fluid decreases as its speed increases piece of paper



the high P wins and the paper flutters upward

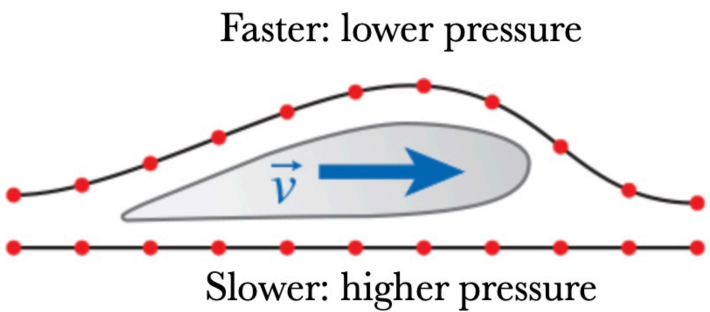


like a straw

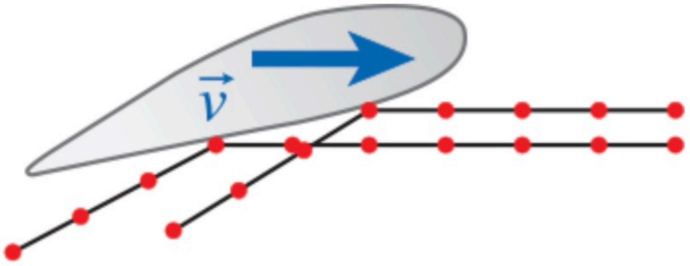
fluid rises up into air stream and is blown out the nozzle

(insert airplane graphic here)

Bernoulli's Principle



Newton's Third Law



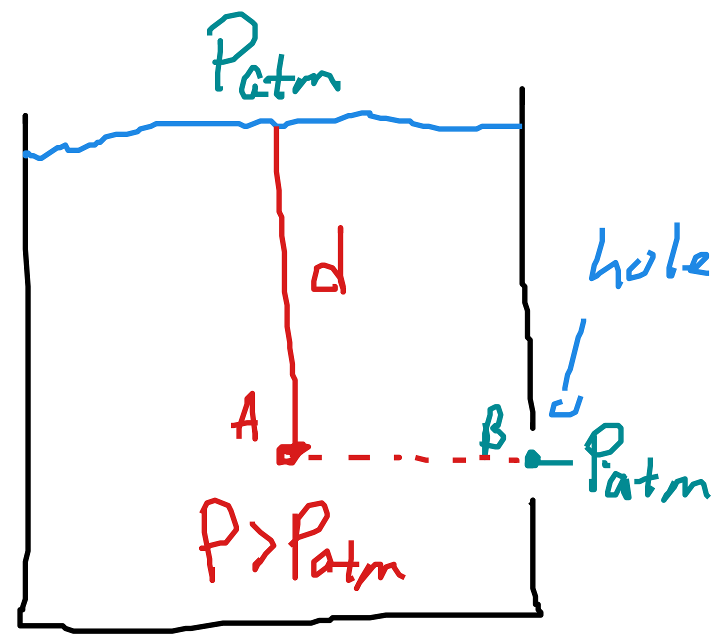
Bernoulli's Equation

depth below reference

$$P = P_0 + \rho_f g d - \frac{1}{2}\rho_f V^2$$

pressure in a fluid

reference pressure



$$P_A = P_{atm} + \rho_f g d$$

$$P_B = P_{atm} + \rho_f g d - \frac{1}{2}\rho_f V^2$$

but

$$P_B = P_{atm}$$ open to air

therefore

$$\cancel{\rho_f} g d - \frac{1}{2}\cancel{\rho_f} V^2 = 0$$

$$g d = \frac{1}{2} V^2$$

$$V^2 = 2 g d$$

$$V = \sqrt{2 g d}$$