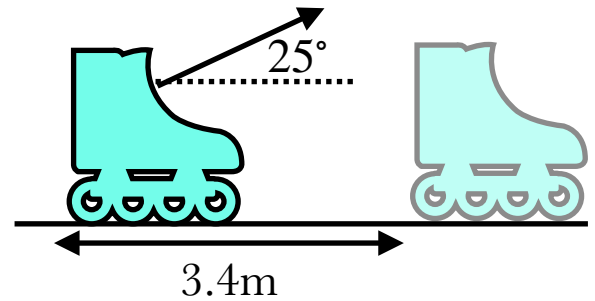


Week 5: Energy

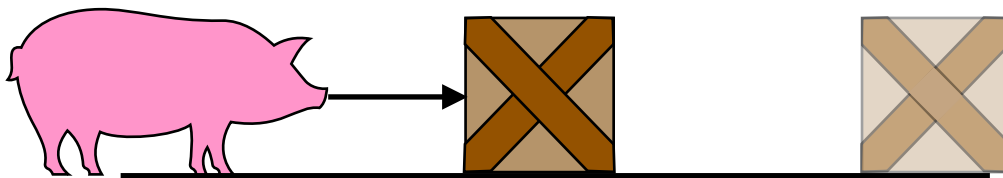
Additional Homework

1. A rollerskate is dragged across the ground for 3.4 meters, by someone pulling on its shoelace. The shoelace makes a 25° angle with the horizontal, and the tension in the shoelace is 5.2N. What is the work done on the rollerskate by the shoelace? Be sure to include whether it is positive or negative.



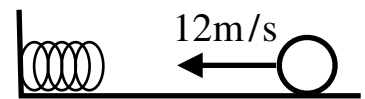
The work done by a force is equal to $W = Fd \cos \theta$. The force in this case is $F = 5.2\text{N}$, and the distance the rollerskate moves is $d = 3.4\text{m}$. The angle θ is between the direction of the force and the motion, which in this case is horizontal. Thus $\theta = 25^\circ$, and so the work done by this force is $W = (5.2)(3.4)\cos 25^\circ = \mathbf{16\text{J}}$.

2. A helpful pig pushes a crate across the barn floor. Initially, the crate has a kinetic energy of $E_i = 20\text{J}$. The pig applies a steady force of 8N on the box, and the kinetic frictional force of the floor on the box is 5N. Once the box has moved 3m to the right, what is the kinetic energy of the box?



The final energy is equal to $E_f = E_i + W$, where W is the total work on the crate from both forces. In this case, the pig does positive work $W_{pig} = (8\text{N})(3\text{m}) = +24\text{J}$, while the kinetic friction does negative work $W_{kfr} = -(5\text{N})(3\text{m}) = -15\text{J}$, and so the total work is $W = 24 - 15 = +9\text{J}$. Thus the final energy is $E_f = E_i + W = 20\text{J} + 9\text{J} = \mathbf{29\text{J}}$.

1. A 5kg ball is rolling with a speed of 12m/s towards a spring with a spring constant of $k = 3000\text{N/m}$. The ball hits the spring and compresses it, causing the ball to stop for a moment. How much does the spring compress?



Initially, the spring is relaxed and the ball is moving.

Finally, the spring is compressed and the ball is not moving.

There is no change in height, so gravitational potential energy doesn't matter.

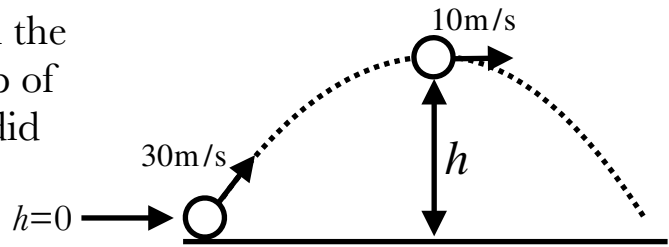
There are no forces acting on the ball-spring system, so external work is zero.

E_f	E_i	W
$E_K = 0$ $E_S = \frac{1}{2}k(\Delta L)^2 = 1500(\Delta L)^2$	$E_K = \frac{1}{2}(5)(12)^2$ $E_S = 0$	
$E_f = 1500(\Delta L)^2$	$E_i = 360\text{J}$	$W = 0$

$$1500(\Delta L)^2 = 360 + 0$$

$$(\Delta L)^2 = \frac{360}{1500} \implies \Delta L = \sqrt{\frac{360}{1500}} = \mathbf{0.49\text{m.}}$$

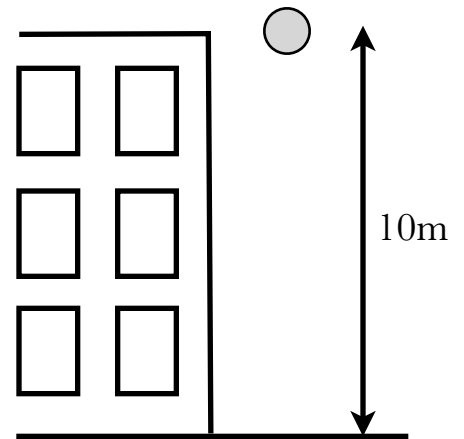
2. A 4kg ball is launched into the air from the ground with a speed of 30m/s. At the top of its arc, it is moving at 10m/s. How high did the ball go? Let ground level be $h=0$.



Initially, the ball has no E_G because it is at $h = 0$, but it does have kinetic energy. At the top of its arc, the ball has both kinetic and gravitational energy. No work is done on the ball because the only force on the ball is weight (which doesn't count since we're already using gravitational energy.)

E_f	E_i	W
$E_K = \frac{1}{2}(4)(10)^2 = 200$	$E_K = \frac{1}{2}(4)(30)^2 = 1800$	0
$E_G = 4(9.8)h = 39.2h$	$E_G = 0$	
$E_f = 200 + 39.2h$	$E_i = 1800$	
$200 + 39.2h = 1800 + 0 \implies h = \frac{1600}{39.2} = \mathbf{40.8m}$		

3. A 2kg ball is dropped from the top of a 10 meter building. How fast will the ball be moving *right before* it hits the ground?

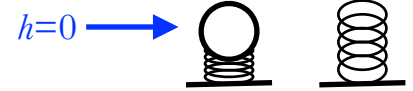


If we measure height from the top of the building, then the ball has no kinetic or gravitational potential energy at first. At the bottom, it has kinetic energy and negative gravitational potential energy. The ball is in free fall so no work is done on the ball.

E_f		E_i		W
$E_G = (2)(9.8)(-10) = -196$		$E_G = 0$		0
$E_K = \frac{1}{2}(2)v^2 = v^2$		$E_K = 0$		
$E_f = v^2 - 196$		$E_i = 0$		

$$v^2 - 196 = 0 \implies v = \sqrt{196} = \mathbf{14m/s}$$

4. A 0.8kg ball is launched into the air by a spring with $k = 8000\text{N/m}$ that has been compressed by 0.07m. As it rises into the air, **air resistance does -5J of work** on the ball. How high does the ball go?



Initially, the ball is not moving, and has no gravitational energy, but the spring is compressed. Finally, the ball is also not moving (it's at its highest point) and the spring is relaxed, but it does have gravitational energy. The work done on the ball is -5J .

E_f	E_i	W
$E_G = (0.8)(9.8)h$	$E_G = 0$	$W = -5$
$E_K = 0$	$E_K = 0$	
$E_s = 0$	$E_s = \frac{1}{2}(8000)(0.07)^2 = 19.6$	
$E_f = 7.84h$	$E_i = 19.6$	

$$7.84h = 19.6 - 5 \implies h = \frac{14.6}{7.84} = \mathbf{1.86\text{m}}$$