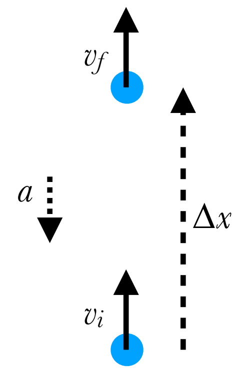
Solving Constant-Acceleration Problems in One Dimension

1) Determine the initial and final events: these should be times when you are either given some piece of information or when you want to determine some information.



2) Draw the object at the initial and final events in the appropriate relative positions, with vectors drawn showing the initial and final velocities (attached to the object), the acceleration (as a separate free-floating arrow), and the displacement (an arrow from the initial location to the final location). If you don't know the direction of a velocity or acceleration, draw the arrow in the positive direction and label it with the appropriate variable.

3) Define the positive direction.

4) Make a table of the five variables displacement Δx, initial velocity v_i, final velocity v_f, acceleration a, and time interval Δt. Fill in the variables you know (and convert to SI). You need at least <u>three</u>.

5) Decide what variable you need to solve for, and write NEED next to it.

6) Choose an equation from the ones on the right. If there is a variable that you don't know and don't need, choose the equation that does not include that variable. If not, choose the equation that includes the needed variable that looks easiest to solve.

$$v_f = v_i + a(\Delta t) \qquad (no\ \Delta x)$$

$$\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2 \qquad (no\ v_f)$$

$$\Delta x = v_f(\Delta t) - \frac{1}{2}a(\Delta t)^2 \qquad (no\ v_i)$$

$$v_f^2 = v_i^2 + 2a(\Delta x) \qquad (no\ \Delta t)$$

$$\Delta x = \frac{1}{2}(v_i + v_f)(\Delta t) \qquad (no\ a)$$

7) Solve the equation for the needed variable.

8) Determine whether the result is sensible. If there are two answers, choose the one that makes the most sense.

Filling in the Variables

In many cases, determining which of the five variables are known is pretty easy. "The initial velocity of the car is 6m/s" doesn't leave much room for guesswork.

There are some phrases, however, which tell us something about the motion, but which aren't quite so obvious. Here are a few:

- **What it says:** "The car starts from rest."
  What it means: The initial velocity is zero.
- **What it says:** "When the ball reaches its highest point."
  What it means: When the velocity is zero.
  Why: *The ball is at a turning point, a maximum or minimum in its position. As we saw in our discussion of graphs, the velocity is zero at turning points.*
- **What it says:** "I throw a ball into the air and it returns to my hand."
  What it means: The displacement is zero.
  Why: *The displacement* Δy *is the distance between the starting point and the ending point. In this case, the ball starts and ends in my hand, and so the displacement is zero.*
- **What it says:** "when the book hits the ground"
  What it means: "*the moment before* the book hits the ground"
  Why: *Collisions are complicated things which the tools we've developed in this chapter are not equipped to handle. Since we can't deal with what happens between the moment the book makes contact with the ground and the moment the book stops, we must be content to ask what happens right before the book hits the ground.*
- Also remember that the acceleration is related to the forces: if youknow the net force, then the acceleration is $a = F_{net}/m$. In particular, an object in free fall has an acceleration of $a = mg/m = g = 9.8\text{m/s}^2$ downward.

Steps for Solving 2D Constant-Acceleration Problems

1) Determine the initial and final events.

2) Draw a picture: draw the object at the initial and final events in the appropriate positions, with vectors drawn showing the initial and final velocities. Draw a separate vector for the acceleration, and the horizontal and vertical components of the displacement vector.

Δx	Δy
v_{ix}	v_{iy}
v_{fx}	v_{fy}
a_x	a_y
Δt	

3) Define the positive *x* and *y* directions.

4) Fill out a 2D table like the one on the right with what you're given and what you need.

If you are given a vector like $\vec{v}_i$, you can usually fill in both columns of that row. If the vector is at an angle, then use trig functions. A row *rarely* has two identical values unless the quantity is zero.

5) If the variable you need is in a column with <u>three</u> known values, possibly including time, solve for that value directly as if it were a 1D problem.

6) If the variable you need is in a column with only two known values, but the other column has three known values, then solve the other column for time, and then use time to solve for what you need.

6) Determine whether the result is sensible. If there are two answers, choose the one that makes the most sense.

Quadratic Equations

Sometimes when you solve for time in a kinematics problem, you will run into an equation where there is a term containing $(\Delta t)^2$ and a term containing (Δt) in it; this is a quadratic equation and needs to be solved differently from a linear equation:

1. Rearrange the terms in the equation in the order $a(\Delta t)^2 + b(\Delta t) + c = 0$: that is, so that the $(\Delta t)^2$ term is first, the (Δt) term is second, the constant term is third, and it is all equal to zero. It make things a little less confusing if you make sure that a is positive.
2. Identify the coefficients *a*, *b*, and *c*. Make sure you include any minus signs. If there are any terms missing, their coefficient is zero.
3. Insert those coefficients into the quadratic formula:
$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
4. I recommend finding the value of the square root first: then use your calculator to find both solutions, the one where ± is + and the one where ± is −.
5. If the square root is imaginary because the thing inside it is negative, then there are *no solutions*.
6. If one answer is negative and the other is positive, choose the positive value.
7. Otherwise, look for clues in the problem to determine which solution is correct. When in doubt, include both solutions.

Note: You can also get quadratic equations for the initial or final velocity in the form $v_i^2 = \text{stuff}$, *so that* $v_i = \pm\sqrt{\text{stuff}}$. *You don't need the full-blown quadratic formula for those, just remember that the negative answer is a possibility.*