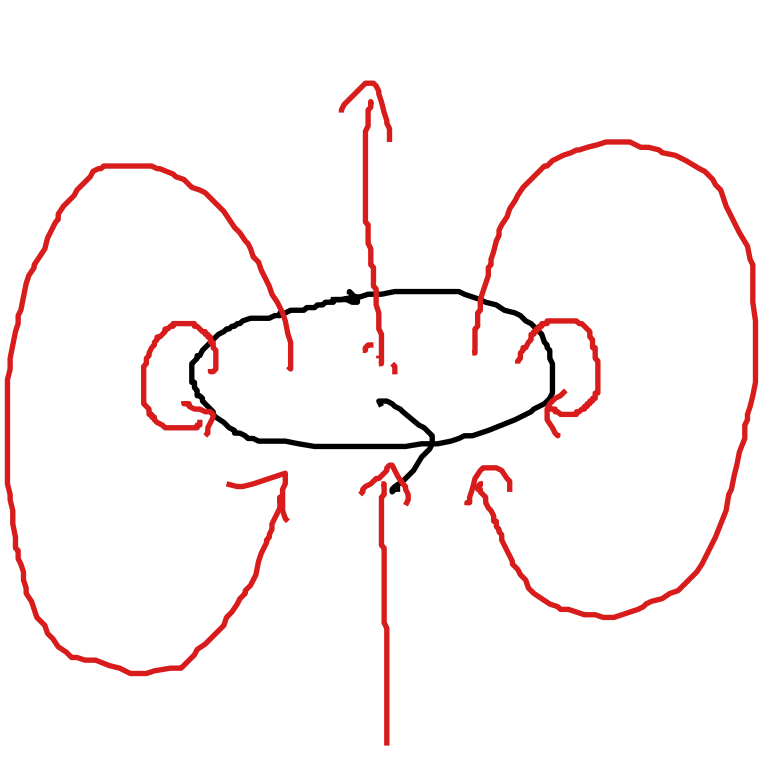
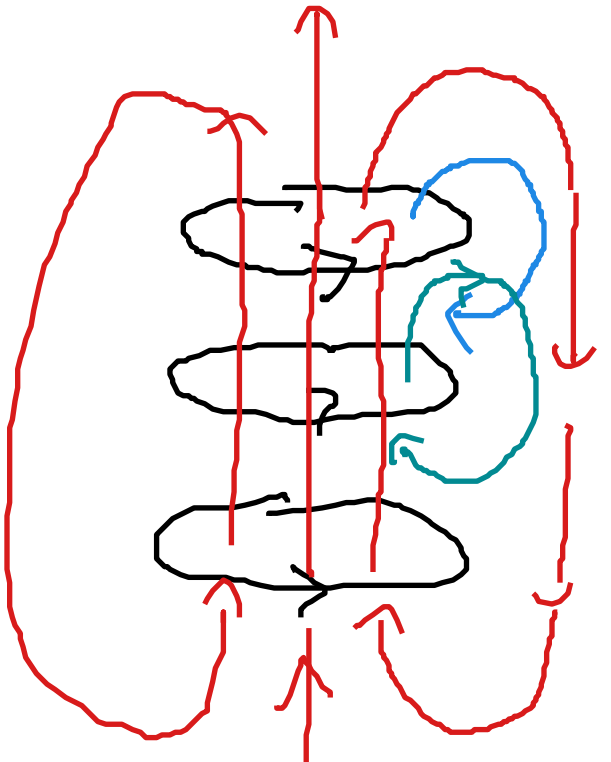
Solenoid

- coil of wire
  around a cylinder

or a stack of loops

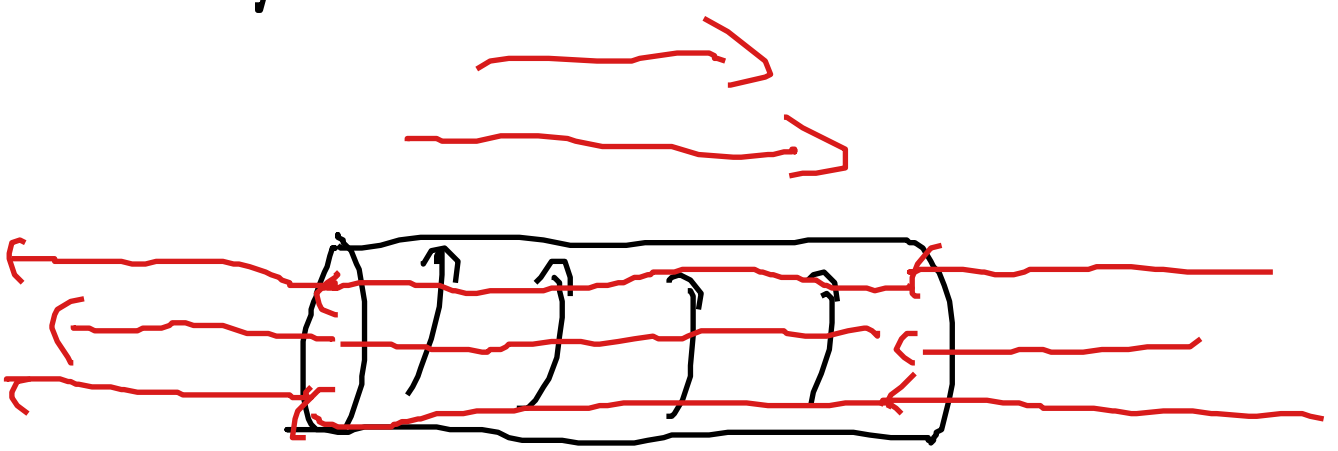




All field lines go through solenoid-
field inside solenoid is strong

Field lines inside solenoid do not converge or diverge (except ends)
so field is uniform

Infinite Solenoid



rotational and translational Symmetry

$\vec{B}$ inside points ←

and can only depend on distance from axis $\vec{B}(r)$

$\vec{B}$ outside points →

and can only depend on distance from axis $\vec{B}(r)$

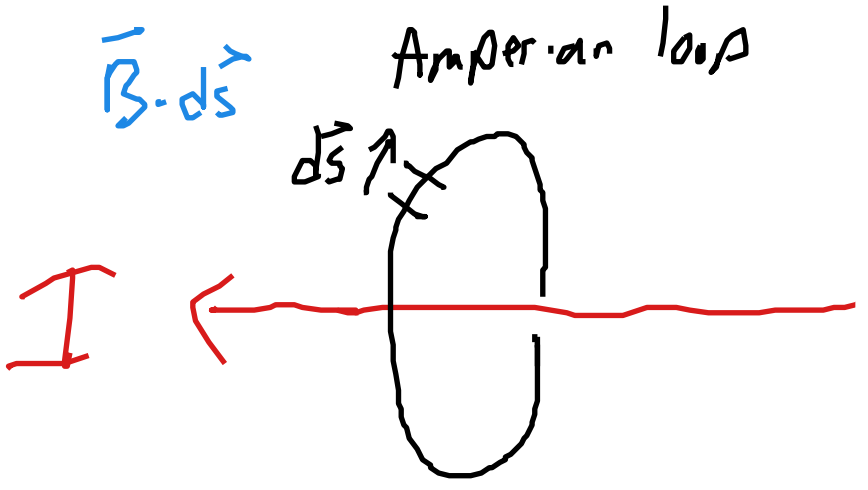
Side-Step

Ampere's Law

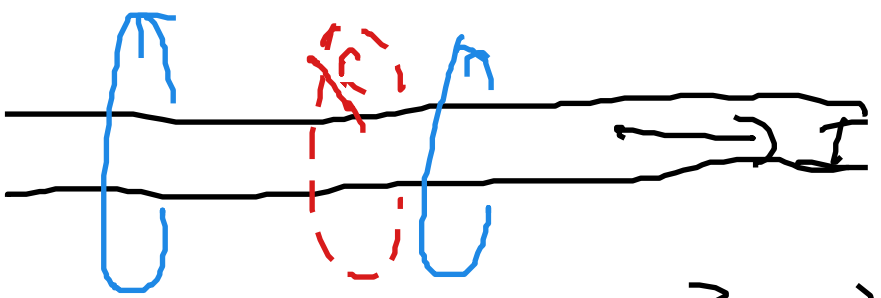
(analogous to Gauss' Law)

choose an Amperian loop

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{thru loop}}$$



Long Straight Wire



Magnetic field is $\vec{B}(r)$ according to symmetry

$\oint \vec{B} \cdot d\vec{s}$

$\vec{B} \times d\vec{s}$ both point around circle

$\oint B(r)\, ds$

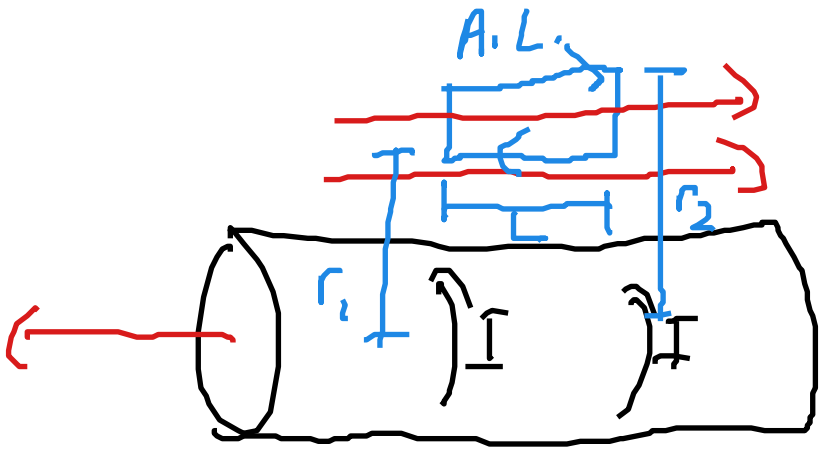
$B(r)$ is same everywhere on the circle due to symmetry

$B(r) \oint ds$

$B(r)\, 2\pi r = \mu_0 I$

$B(r) = \dfrac{\mu_0 I}{2\pi r}$

Solenoid



$$\int \vec{B} \cdot d\vec{s} =$$

top $\int \vec{B} \cdot d\vec{s} = \int B(r_2)\, ds = B(r_2)L$

left $\int \vec{B} \cdot d\vec{s} = 0$

botton $\int \vec{B} \cdot d\vec{s} = -\int B(r_1)\, ds = -B(r_1)L$

right 0

$$\int \vec{B} \cdot d\vec{s} = [B(r_2) - B(r_1)]L = \mu_0 I_{thru}$$

$B(r_1) = B(r_2)$ so $[B(r_2) - B(r_1)] = 0$

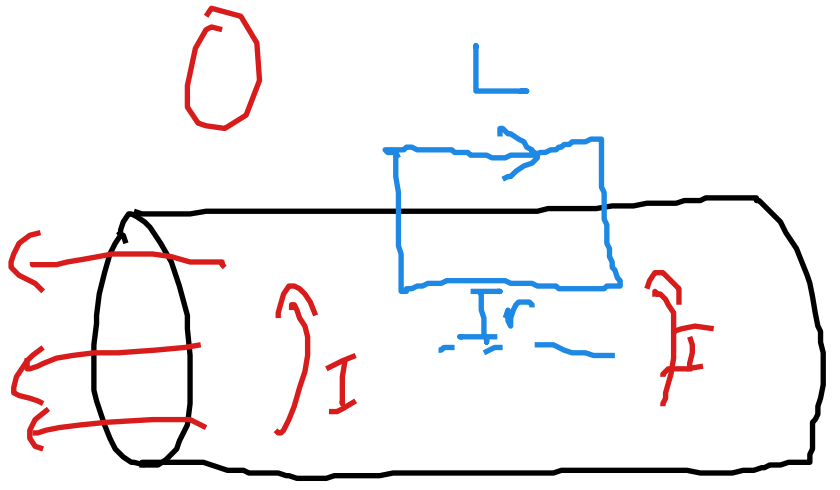
$= 0$ no wires piercing A.L.

True for any r_1, r_2 outside the solenoid

$B = 0$ at $r = \infty$

$\therefore$ $B = 0$ everywhere outside the ∞ solenoid

it does not leak



$$\int \vec{B} \cdot d\vec{s} = \underbrace{0}_{\text{top}} + \underbrace{0}_{\text{left}} + \underbrace{B(r)L}_{\text{bottom}} + \underbrace{0}_{\text{right}}$$

no field

$$B(r)L = \mu_0 I_{thru}$$

$$= \mu_0 N I$$

N = # of coils passing through loop

$$B(r) = \mu_0 I \frac{N}{L}$$

$$= \mu_0 I n$$

n: coil density

coils / meter

For a strong field,

- turn up current
- use thinner wires to pack them closer together

(but don't forget to insulate them!)

If solenoid is wrapped around a ferromagnetic core, replace μ_0 with μ (Fe: $\mu = 5000\mu_0$)

$$B = \mu I n$$