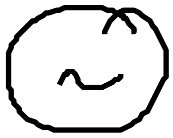
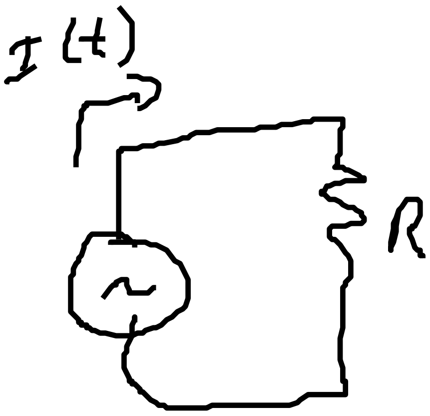
 $\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t)$

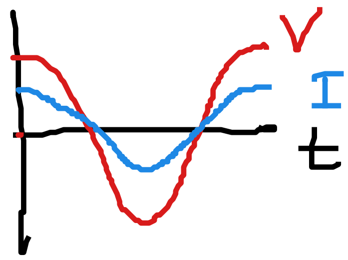


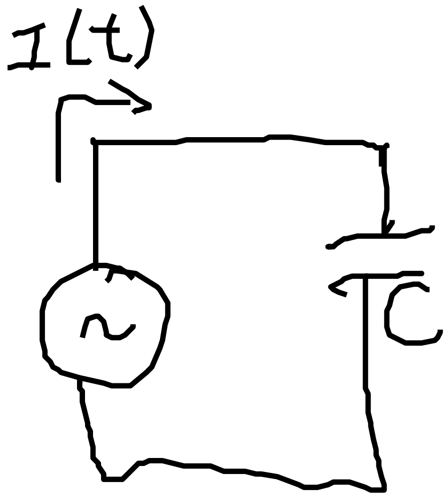
$$I(t) = I_0 \cos(\omega t + \varphi)$$

(general)

$$I(t) = \frac{\mathcal{E}(t)}{R} = \frac{\mathcal{E}_0}{R} \cos(\omega t)$$

In sync





$\Delta V = \frac{Q}{C}$

$$Q = \int I \, dt$$

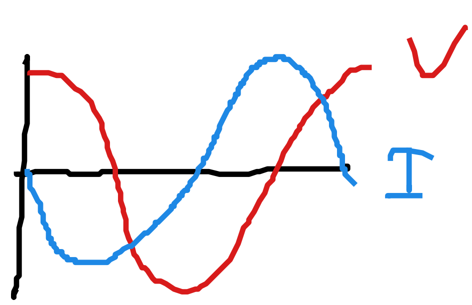
$$I(t) = -\omega C \mathcal{E}_0 \sin \omega t$$

$$I(t) = -\frac{\mathcal{E}_0}{X_c} \sin \omega t$$

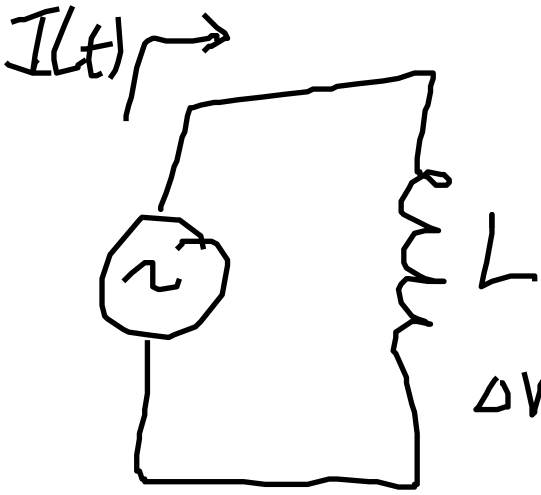
X_c: reactance of capacitor

big X_c, small current

$$X_c = \frac{1}{\omega C}$$



current leads voltage



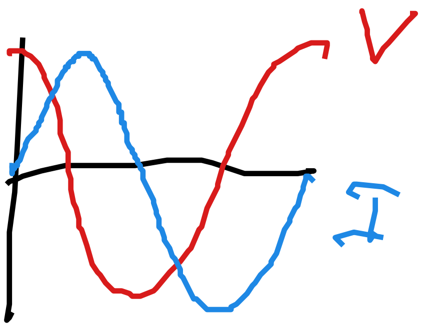
$\Delta V = -L\frac{dI}{dt}$

$$I(t) = \frac{\mathcal{E}_0}{\omega L}\sin\omega t$$

$$= \frac{\mathcal{E}_0}{X_L}\sin\omega t$$

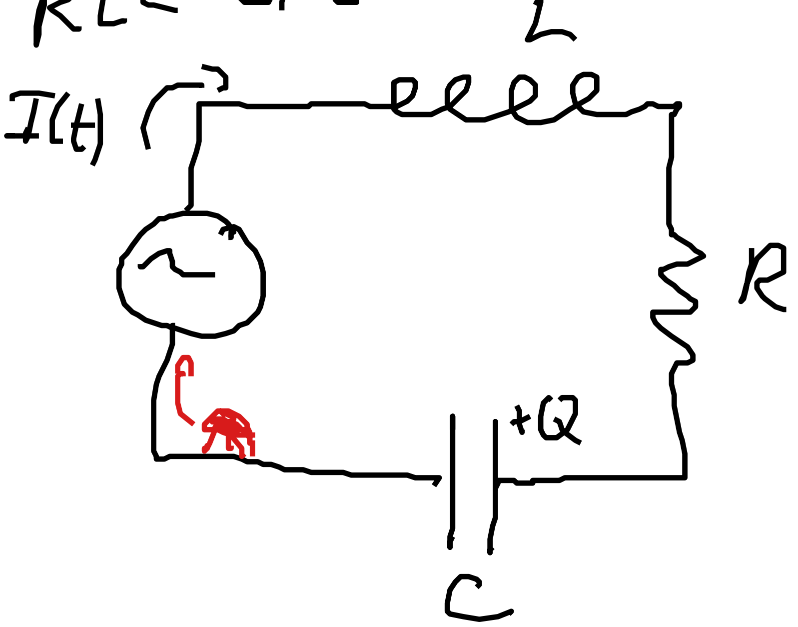
$X_L = \omega L$: reactance of inductor

big X_L, small current



current lags voltage

RLC Circuit



$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t - \phi)$$

$$I(t) = I_0 \cos(\omega t)$$

$$\frac{dI}{dt} = -I_0 \omega \sin \omega t$$

$$Q = \int I \, dt = \frac{I_0}{\omega} \sin(\omega t)$$

$$\mathcal{E}(t) - L\frac{dI}{dt} - IR - \frac{Q}{C} = 0$$

$$\mathcal{E}(t) = L\frac{dI}{dt} + IR + \frac{Q}{C}$$

$$\mathcal{E}_0 \cos(\omega t - \phi) = -I_0 L\omega \sin \omega t + I_0 R \cos \omega t + \frac{I_0}{\omega C} \sin \omega t$$

$$\frac{\mathcal{E}_0}{I_0} \cos(\omega t - \phi) = R\cos \omega t + \left(\frac{1}{\omega C} - L\omega\right) \sin \omega t$$

$$= R \cos \omega t + (X_C - X_L) \sin \omega t$$

$$Z = \frac{\mathcal{E}_0}{I_0}$$ impedance of circuit

$$I_0 = \frac{\mathcal{E}_0}{Z}$$ big Z, small current

$$\cos(\omega t - \phi) = \cos\omega t \cos\phi + \sin\omega t \sin\phi$$

$$Z \cos\omega t \cos\phi + Z \sin\omega t \sin\phi$$

$$= R\cos\omega t + (X_C - X_L)\sin\omega t$$

$$Z\cos\phi = R \qquad Z\sin\phi = X_C - X_L$$

$$Z^2\cos^2\phi + Z^2\sin^2\phi = R^2 + (X_C - X_L)^2$$

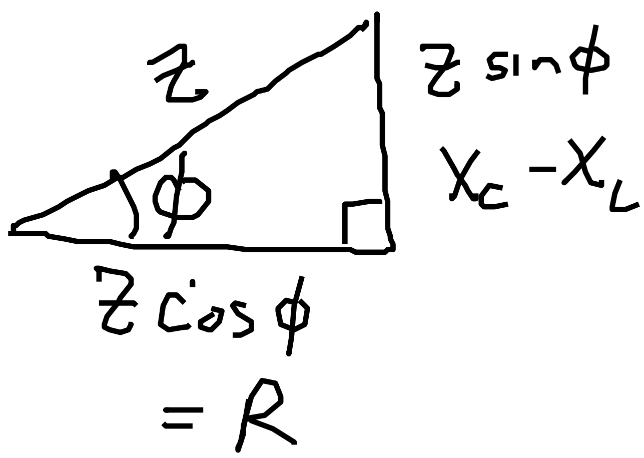
$$Z^2 = R^2 + (X_C - X_L)^2$$

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

$$\frac{\cancel{Z}\sin\phi}{\cancel{Z}\cos\phi} = \frac{X_C - X_L}{R}$$

$$\boxed{\tan\phi = \frac{X_C - X_L}{R}}$$

ϕ: how much out of sync is I from V



To maximize current, minimize impedance $\quad I_0 = \frac{\mathcal{E}_0}{Z}$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z(\omega) = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

to minimize $Z(\omega)$

choose ω so $X_C = X_L$

$$\frac{1}{\omega C} = \omega L \rightarrow \omega^2 = \frac{1}{LC}$$

$$\rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonant frequency

$\phi = 0$

current is in sync with the voltage

Power output by resistor in an RLC circuit

$$P(t) = I(t)^2 R$$

$$\langle P \rangle = \langle I^2 \rangle R$$

$$= \frac{1}{2} I_0^2 R$$

I_0: max current or peak current

$$P_{avg} = I_{rms}^2 R$$

$$I_{rms} = \sqrt{\langle I^2 \rangle}$$

$$= \sqrt{\frac{1}{2} I_0^2}$$

$$I_{rms} = \frac{1}{\sqrt{2}} I_0$$

$$\approx 0.7\, I_0$$

$$Z = \frac{V_0}{I_0}$$

$$V_{rms} = \frac{1}{\sqrt{2}} V_0$$

$$Z = \frac{V_{rms}}{I_{rms}} \rightarrow I_{rms} = \frac{V_{rms}}{Z}$$

$$\langle P \rangle = I_{rms}^2 R$$

$$= I_{rms} I_{rms} R$$

$$= \frac{V_{rms}}{Z} I_{rms} R$$

average power output by resistor

$$= I_{rms} V_{rms} \frac{R}{Z}$$

power factor

$$= I_{rms} V_{rms} \cos\phi$$