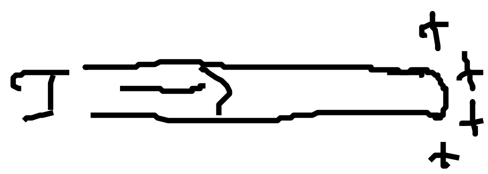
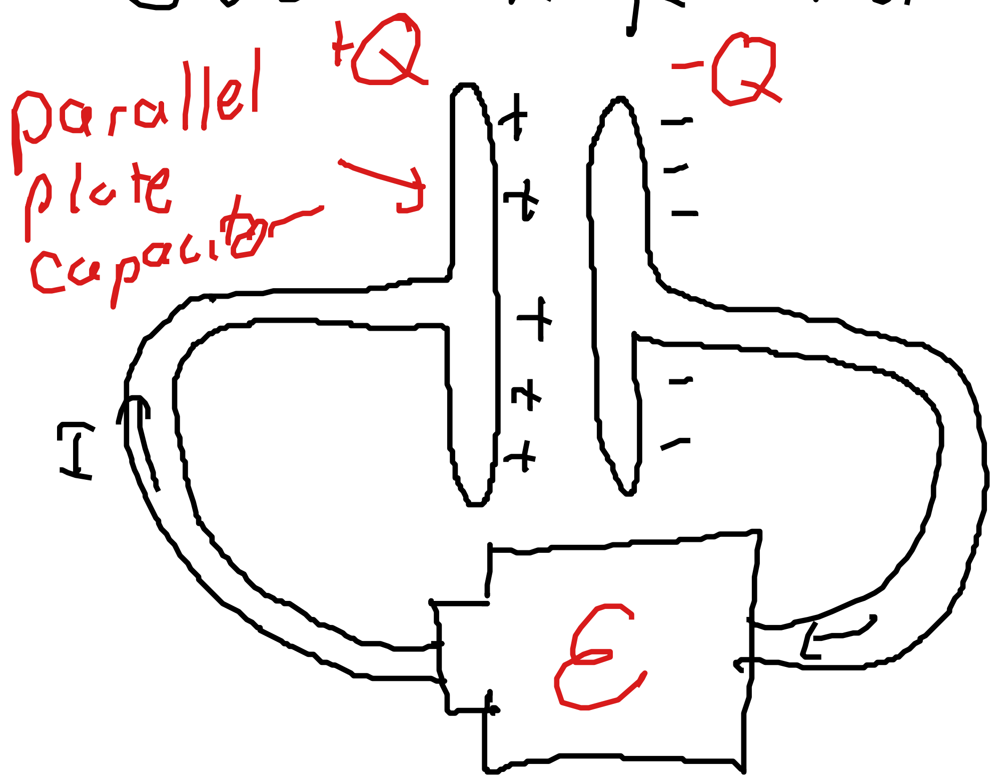


No steady current possible because of buildup of charge.

Give charge more space



takes longer for current to stop

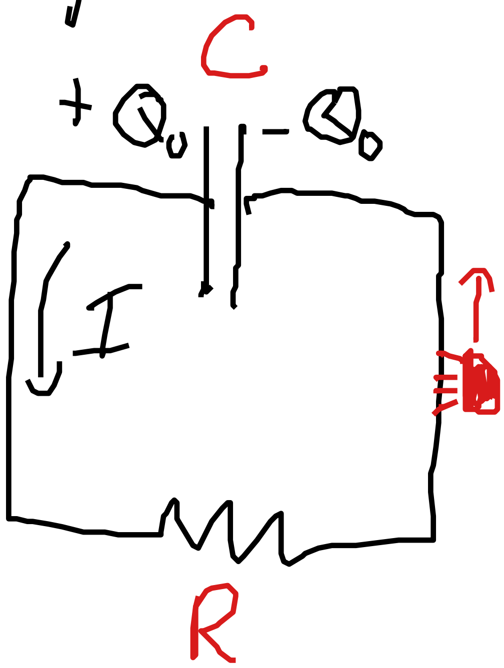
Capacitor charges from the battery

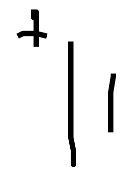
until $Q = C\mathcal{E}$

then it stops

RC Circuits

- Discharge a capacitor





Let $Q(t)$ be the charge on positive plate.

$$+\overbrace{\frac{Q(t)}{C}}^{\Delta V_c} - I(t)R = 0$$

$$\frac{dQ}{dt} = -I$$

$$\frac{Q(t)}{C} + \frac{dQ}{dt} R = 0$$

First-Order Differential Equation

$$\frac{dQ(t)}{dt} = -\frac{Q(t)}{RC}$$

$$Q(t) = Ae^{-t/\tau}$$

tau: time constant

A is arbitrary constant

e^x

x must be dimensionless

$$\frac{dQ(t)}{dt} = A\left(-\frac{1}{\tau} e^{-t/\tau}\right)$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

$$-\frac{A}{\tau} e^{-t/\tau} = -\frac{1}{RC} A e^{-t/\tau}$$

$$\cancel{-}\frac{\cancel{A}}{\tau} = \cancel{-}\frac{\cancel{A}}{RC}$$

$$\boxed{\tau = RC}$$

$$R = \frac{\Delta V}{I}$$

$$C = \frac{Q}{\Delta V}$$

Units: $\Omega F = \frac{\cancel{V}}{A} \frac{C}{\cancel{V}} = \frac{\cancel{C}}{\cancel{C}/s} = s$

$$Q(t) = A e^{-t/RC}$$

Discharging a capacitor

Initial condition: $Q(t=0) = A e^{0}$

$$Q_0 = A$$

initial amount of charge

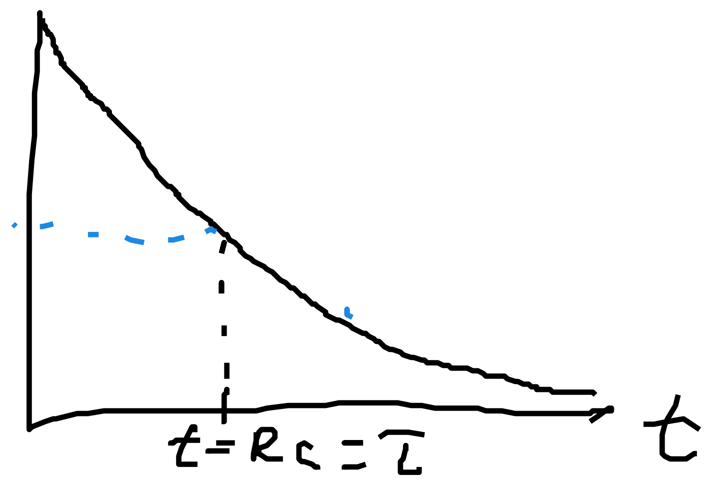
Discharging a capacitor

$$Q(t) = Q_0 e^{-t/RC}$$

$$I(t) = -\frac{dQ}{dt} = -Q_0\left[-\frac{1}{RC} e^{-t/RC}\right]$$

$$I(t) = \underbrace{\frac{Q_0}{RC}}_{I_0} e^{-t/RC}$$

$e^{-t/RC} \longrightarrow$



e

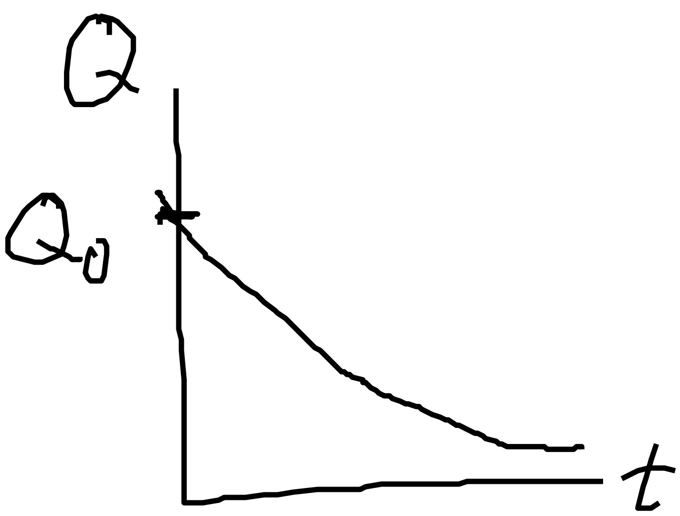
$e^{-RC/RC}$

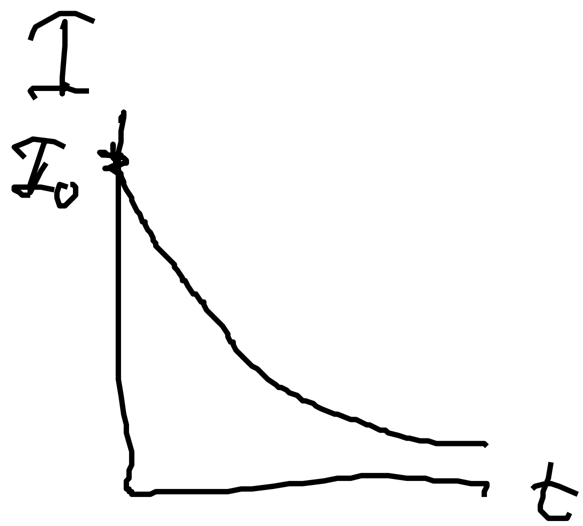
e^{-1}

≈ 0.37

$t=0$: $e^{-t/RC} = 1$

$t \to \infty$: $e^{-t/RC} \longrightarrow 0$



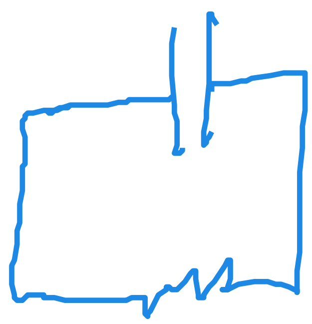


$t=0$: $Q = Q_0$ $\qquad I = \dfrac{Q_0}{RC}$

$t \to \infty$: $Q = 0$ $\qquad I = 0$

τ is time for $e^{-t/\tau}$ to reach $37\% = \frac{1}{e}$

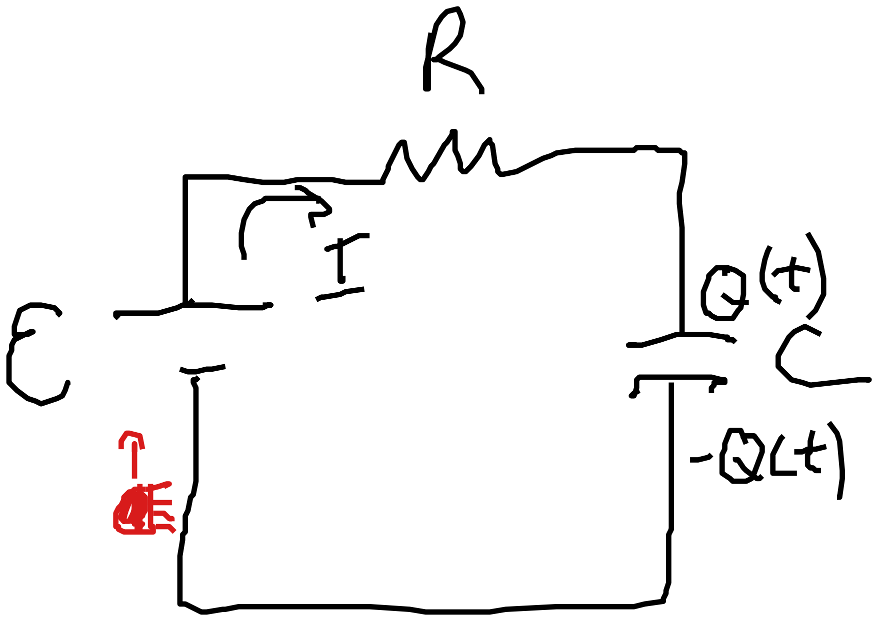
Gives a sense of "how long" it takes to decay



$\tau = RC$

Big C: big τ, holds lots of charge, less pressure to discharge, takes longer

Big R: big τ: current is restricted so takes longer to discharge



↓ down the capacitor from + to −

$$+\mathcal{E} - IR - \frac{Q}{C} = 0$$

$$I = \frac{\mathcal{E}}{R} - \frac{Q}{RC} \qquad \frac{dQ}{dt} = +I$$

$$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{Q}{RC}$$

$$Q(t) = A\left(1 - e^{-t/\tau}\right)$$

$$= A - Ae^{-t/\tau}$$

$$\frac{dQ}{dt} = -A\left(-\frac{1}{\tau}e^{-t/\tau}\right)$$

$$= \frac{A}{\tau}e^{-t/\tau} = I$$

$$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{Q}{RC}$$

$$\frac{A}{\tau}e^{-t/\tau} = \frac{\mathcal{E}}{R} - \frac{1}{RC}A\left[1-e^{-t/\tau}\right]$$

Get all "t" stuff to one side

$$\underbrace{\left[\frac{A}{\tau} - \frac{A}{RC}\right]}_{0}e^{-t/\tau} = \underbrace{\frac{\mathcal{E}}{R} - \frac{A}{RC}}_{0}$$

can only be true for all time if $= 0$

$$\frac{A}{\tau} - \frac{A}{RC} = 0 \longrightarrow \tau = RC$$

$$\frac{\mathcal{E}}{\cancel{R}} - \frac{A}{\cancel{R}C} = 0 \longrightarrow A = C\mathcal{E}$$

Charging a capacitor

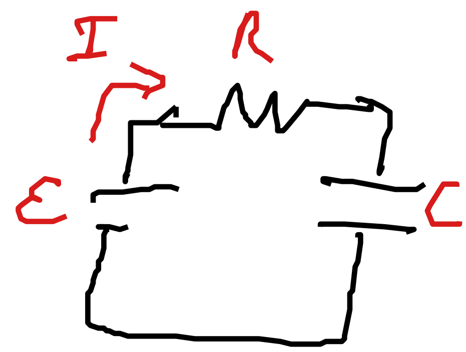
$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{\cancel{C}\mathcal{E}}{R\cancel{C}} e^{-t/RC}$$

$$= \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$t = 0: \; Q(0) = C\mathcal{E}(1 - 1) = 0$$

$$I(0) = \frac{\mathcal{E}}{R}(1) = \mathcal{E}/R$$



empty capacitor acts like a wire

$$t = \infty: \quad Q(\infty) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

$$I(\infty) = \frac{\mathcal{E}}{R}(0) = 0$$

full capacitor acts like a break in wire

$$Q = C\mathcal{E} = C\Delta V$$

what we would've said before