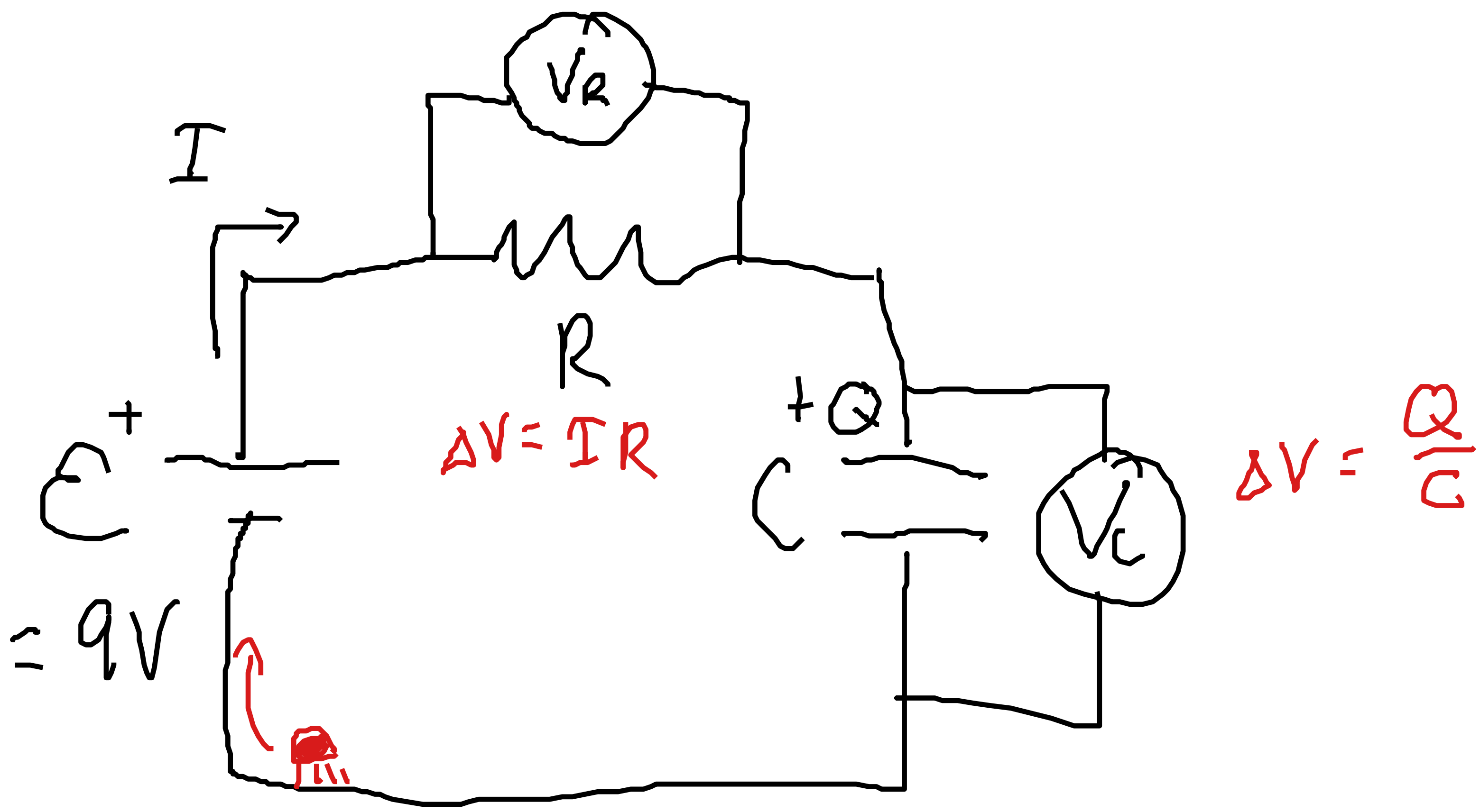
RC Circuits



At time $t = 0$,
capacitor is empty. $Q = 0$

there's a current

$\Delta V_R \neq 0$

cap is empty

$\Delta V_C = 0$

$\Delta V_R = 9V = \mathcal{E}$

$+9 - \Delta V_R + 0 = 0$

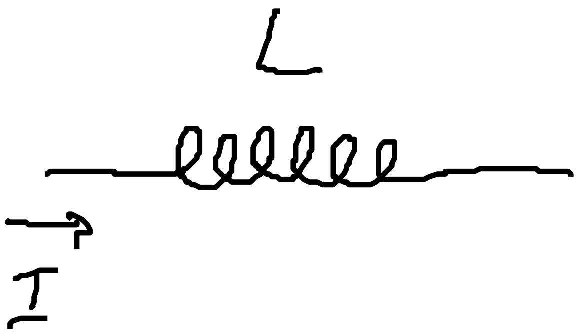
At time $t = \infty$,

$$\Delta V_R = \underset{\text{tricks to a stop}}{\overset{\downarrow}{I}} R = 0 \qquad \Delta V_C = \frac{Q}{C} \neq 0$$

$$\Delta V_C = 9V$$

Inductor

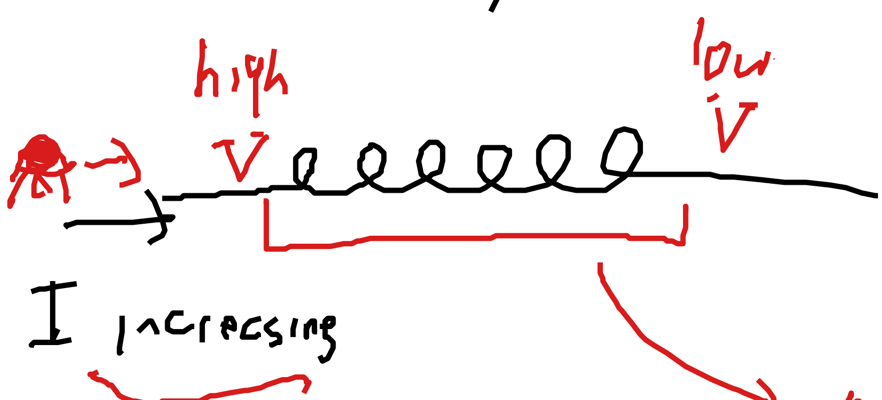
(Solenoid)



If I is steady, inductor is just a wire

If I is increasing

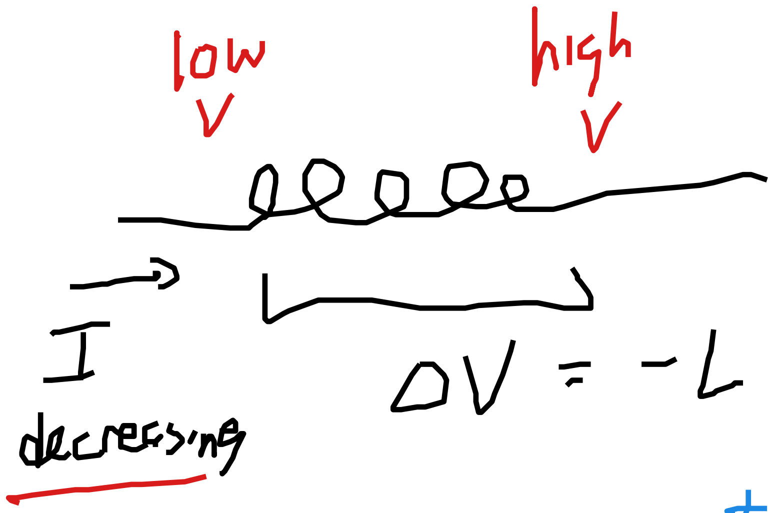
inductor tries to create an induced current in the opposite direction to fight it : acts like a resistor



$$\Delta V = -L\frac{dI}{dt}$$

$\Delta V < 0$

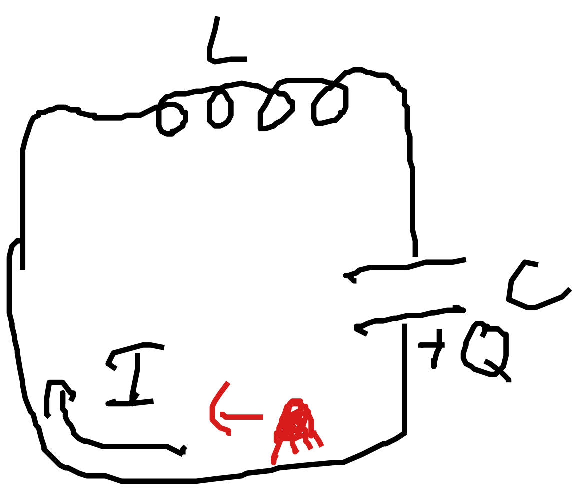
like a resistor



$$\Delta V = -L\frac{dI}{dt} > 0$$

$$\frac{dI}{dt} < 0$$

act like a battery giving the current a boost



LC Circuit

C starts filled

$$-L\frac{dI}{dt} + \frac{Q}{C} = 0$$

$$I = -\frac{dQ}{dt}$$

$$\frac{Q(t)}{C} = L\frac{dI(t)}{dt}$$

$$= L\frac{d}{dt}\left(-\frac{dQ}{dt}\right)$$

$$\frac{Q(t)}{C} = -L\frac{d^2Q(t)}{dt^2}$$

second-order differential equation

- two arbitrary constants
- If you can guess a solution you're right

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q(t)$$

arbitrary

$$Q(t) = A\cos(\omega t) + B\sin(\omega t)$$

OR (better)

must be solved for

$$Q(t) = \underline{A}\cos(\omega t + \underline{\phi_0})$$

arbitrary

$$\frac{dQ}{dt} = -A\omega \sin(\omega t + \phi_0)$$

$$\frac{d^2Q}{dt^2} = -A\omega^2 \cos(\omega t + \phi_0)$$

$$\underbrace{-A\omega^2}\cos(\omega t + \phi_0) = \underbrace{-\frac{1}{LC}A}\cos(\omega t + \phi_0)$$

$$+\cancel{A}\omega^2 = +\frac{1}{LC}\cancel{A}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$Q(t) = A\cos\left(\frac{t}{\sqrt{LC}} + \phi_0\right)$$

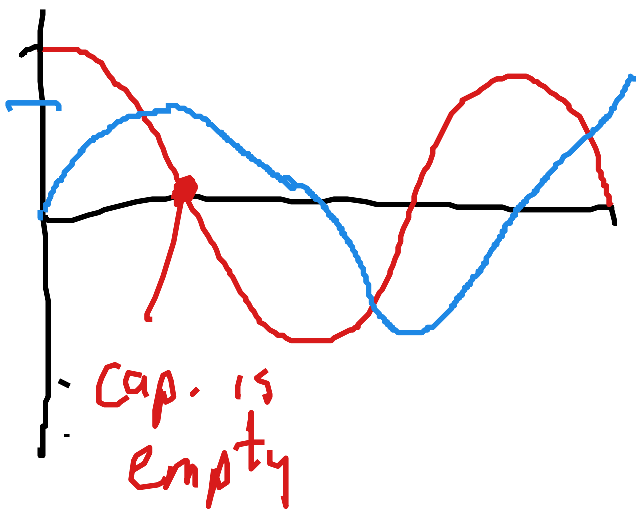
$\phi_0 = 0$ for moment

angular frequency $\omega = 2\pi f$ ← frequency

f = # of oscillations per second

$Q_0 = A$

$I_{max} = \frac{Q_0}{\sqrt{LC}}$



$$I(t) = -\frac{dQ}{dt}$$

$$= Q_0\,\omega\,\sin(\omega t + \phi_0)$$

When Q is maximum, $I = 0$

When $Q = 0$, current "should" stop but inductor won't let it

current keeps flowing & capacitor charges in opposite direction

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

frequency of oscillation $f = \frac{1}{2\pi\sqrt{LC}}$

Big C: small frequency
(takes time to discharge)

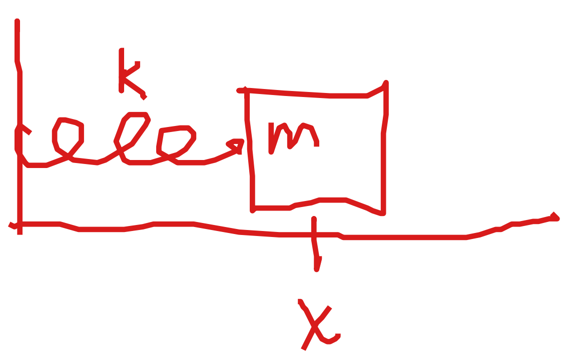
Big L: small frequency
inductor restricts current
& keeps it going

$$L\frac{d^2Q}{dt^2} + \frac{1}{c}Q = 0$$

analogy with $Q \longleftrightarrow x$ position

$$L\frac{d^3x}{dt^2} + \frac{1}{C}x = 0$$

$$ma = \overbrace{-\underbrace{k}\,x}^{\text{force that depends on } x}$$



$Q \longleftrightarrow x$

$I = \frac{dQ}{dt} \longleftrightarrow v$ velocity

$\frac{d^2Q}{dt^2} \longleftrightarrow a$

L acts like m : inertia

$\frac{1}{c}$ acts like k '