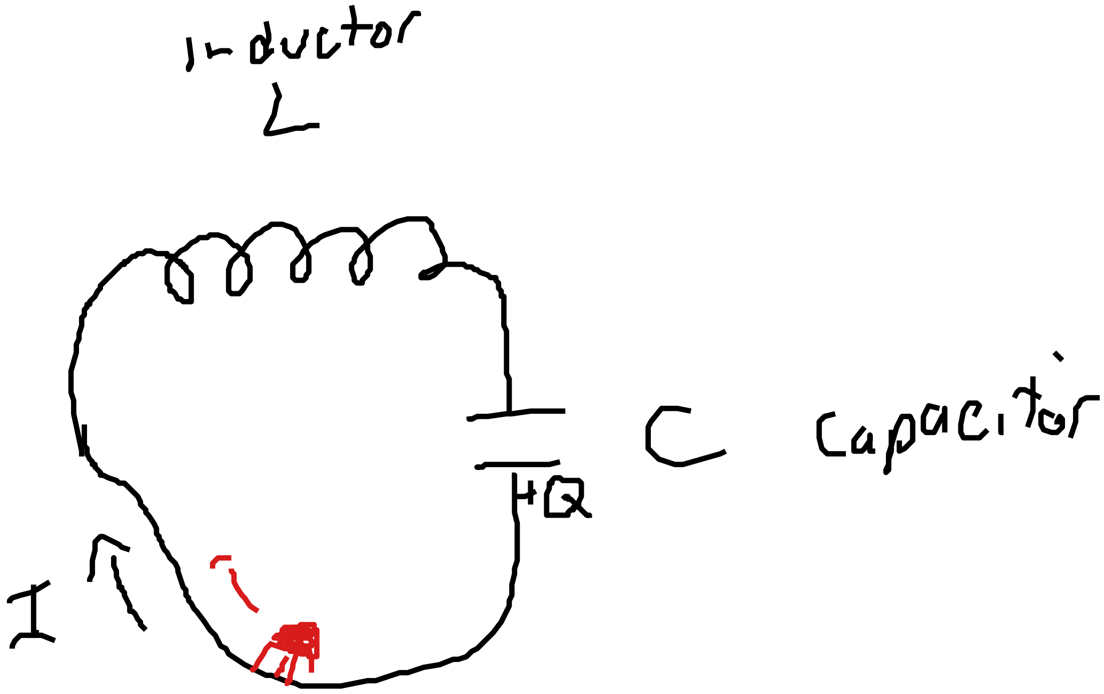



$$I = -\frac{dQ}{dt}$$

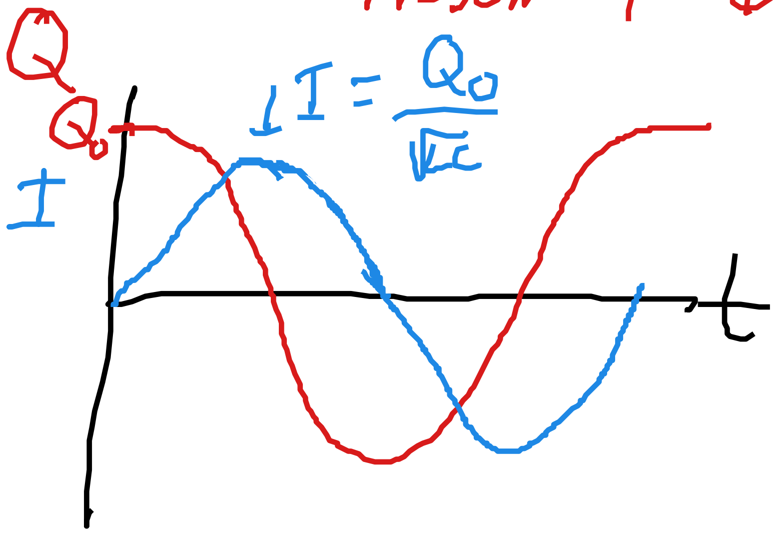
As + current flows, Q decreases

$$-L\frac{dI}{dt} + \frac{Q}{C} = 0$$

$$Q(t) = Q_0 \cos\left(\frac{1}{\sqrt{LC}}t + \phi_0\right)$$

$$I(t) = -\frac{dQ}{dt} = \frac{Q_0}{\sqrt{LC}} \sin\left(\frac{1}{\sqrt{LC}}t + \phi_0\right)$$

Assuming $\phi_0 = 0$



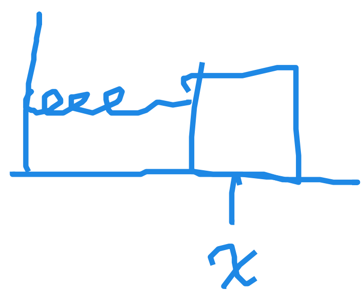

$$\omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Q & I are out-of-phase with each other by $\pi/2$.

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

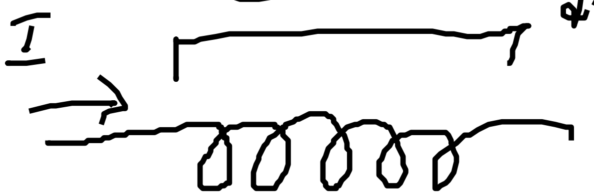
$$m\frac{d^2x}{dt^2} + kx = 0 \rightarrow ma = -kx$$




$$E_C = \frac{1}{2}\frac{Q^3}{C}$$ energy in a capacitor

Where does the energy go when capacitor is empty? into the inductor.

$$\Delta V = -L\frac{dI}{dt}$$




$$P = I\Delta V$$

$$dt \times P = -LI\frac{dI}{dt} \times dt$$

$$\int P\,dt = -L\int I\,dI$$

$$\int \frac{dE}{dt}\,dt = -L\left(\frac{1}{2}I^2\right)$$

− ! energy stolen from current

$$E_L = \frac{1}{2}LI^2$$

+ ! energy stored in inductor

$$E_{tot} = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

$\phi_0 = 0$

$$= \frac{1}{2C}\left[Q_0 \cos(\omega t)\right]^2 + \frac{1}{2}L\left[Q_0\omega \sin(\omega t)\right]^2$$

$\omega = \frac{1}{\sqrt{LC}}$

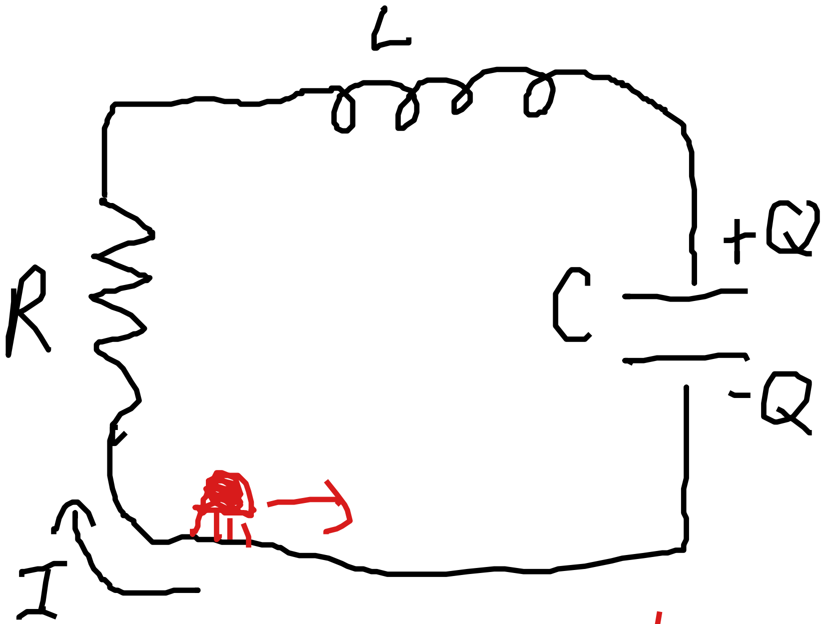
$$\frac{1}{2C}Q_0^2 \cos^2 \omega t + \frac{1}{2}LQ_0^2\omega^2 \sin^2(\omega t)$$

$\frac{1}{LC}$

$$= \frac{1}{2C}Q_0^2 \cos^2 \omega t + \frac{1}{2C}Q_0^2 \sin^2(\omega t)$$

$$= \frac{Q_0^2}{2C}\left[\cos^2 \omega t + \sin^2 \omega t\right]$$

$$\boxed{E_{tot} = \frac{Q_0^2}{2C}}[1] \quad \text{constant}$$




different!

$$I = +\frac{dQ}{dt}$$

upstream in inductor

$$+\frac{Q}{C} + L\frac{dI}{dt} + IR = 0$$

$$\frac{1}{C}Q(t) + L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q(t) = 0$$

$$ma + \sqcup v + kx$$

air resistance (or frictionish) — spring force

$$Q = Q_0 e^{-t/\tau}\cos(\omega t)$$

$$\underbrace{\left[\frac{1}{LC} - \frac{R}{L\tau} + \frac{1}{\tau^2} - \omega^2\right]}_{=0} \cos \omega t$$

$$+ \underbrace{\left[2\frac{\omega}{\tau} - \frac{R\omega}{L}\right]}_{=0} \text{sen}\, \omega t = 0$$

$$\frac{2\omega}{\tau} - \frac{R}{L}\omega = 0$$

$$\frac{2}{\tau} = \frac{R}{L} \rightarrow \tau = \frac{2L}{R}$$

big τ'
slow decay

$$\frac{1}{LC} - \frac{R}{L\tau} + \frac{1}{\tau^2} - \omega^2 = 0$$

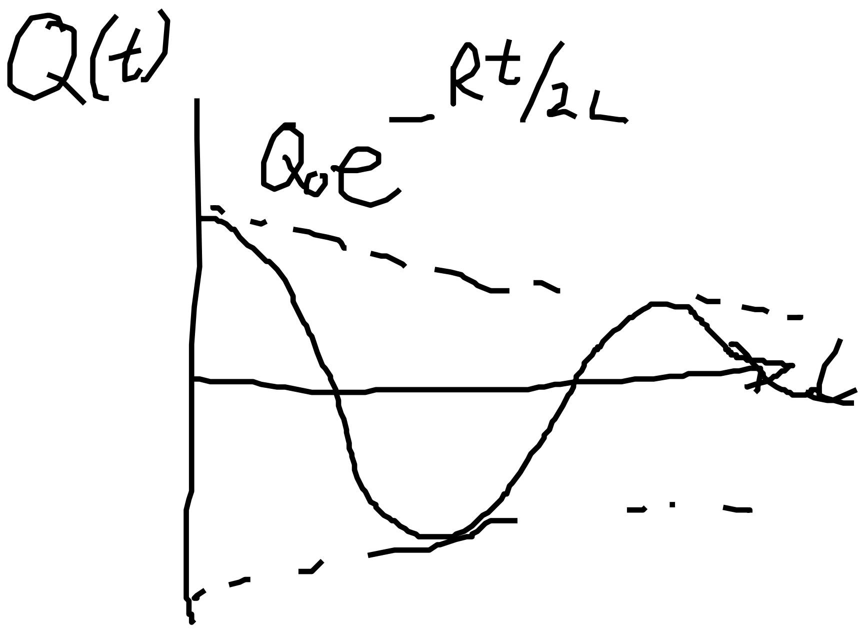
$$\frac{1}{LC} - \frac{R}{L}\frac{R}{2L} + \frac{R^2}{4L^2} - \omega^2 = 0$$

$$\frac{1}{LC} - \frac{R^2}{2L^2} + \frac{R^2}{4L^2} - \omega^2 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

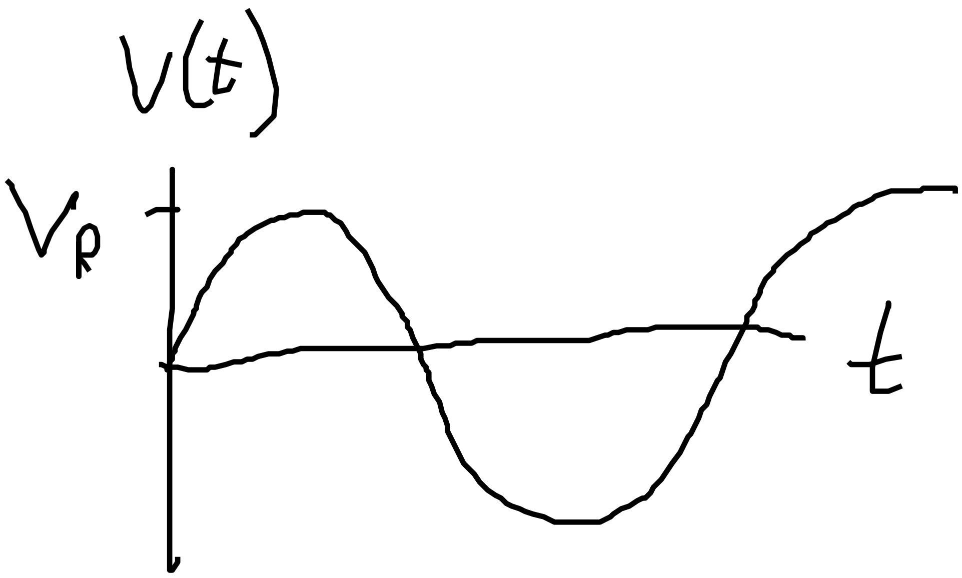
$$\omega^2 = \omega_0^2 - \left(\frac{R}{2L}\right)^2$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$$



# Alternating Current

$V(t)$

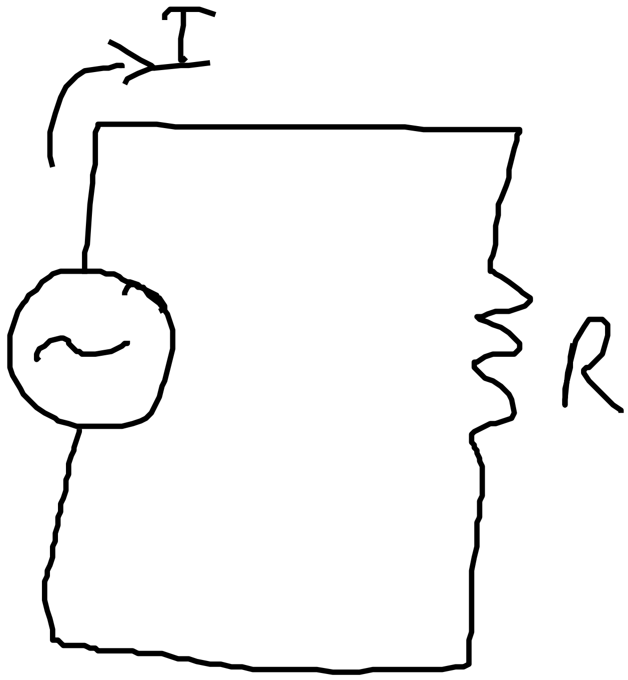



$$V(t) = V_p \cos(\omega t + \phi_0)$$

$V_p$ : peak voltage

$\omega$ : angular frequency of oscillation

$$V(t) = V_p \cos(2\pi f t + \phi_0)$$




$$V(t) = V_p \cos(2\pi f t)$$

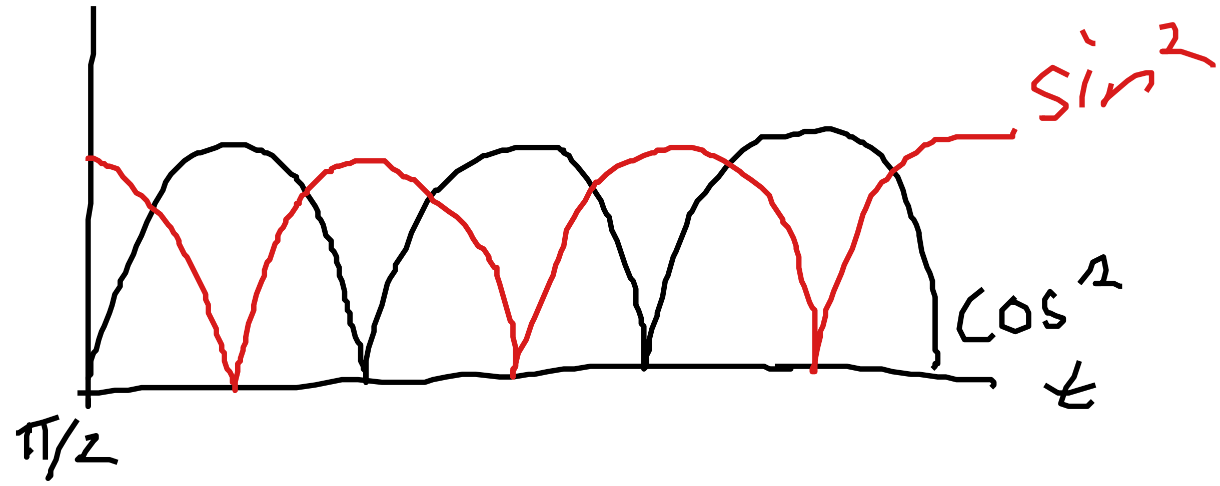
$$I(t) = \frac{V(t)}{R} = \frac{V_p}{R}\cos(2\pi f t)$$

$$P_{\substack{\text{output}\\ \text{by resistor}}} = I^2 R = \frac{V_p^2}{R^2}\cos^2(2\pi f t)\, R$$

$$= \frac{V_p^2}{R}\cos^2(2\pi f t)$$

average

$$\langle P \rangle = \frac{V_p^2}{R}\langle \cos^2 2\pi f t \rangle$$




$$\langle \cos^2 + \sin^2 \rangle = 1$$

$$\langle \cos^2 \rangle + \langle \sin^2 \rangle = 1$$

$$2\langle \cos^2 \rangle = 1$$

$$\langle \cos^2 \rangle = \frac{1}{2}$$

$$\langle P \rangle = \frac{1}{2}\frac{V_p^2}{R}$$

I'd really like to write $\langle P \rangle = \frac{V_?^2}{R}$

$$\frac{1}{2}\frac{V_p^2}{\cancel{R}} = \frac{V_?^2}{\cancel{R}}$$

$$V_?^2 = \frac{V_p^2}{2}$$

$$V_{rms} = \frac{V_p}{\sqrt{2}}$$

what electricians talk about in AC circuits

US: $V_{rms} = 120V$ $f = 60Hz$

$$V_p = 120 \times \sqrt{2} = 170V$$