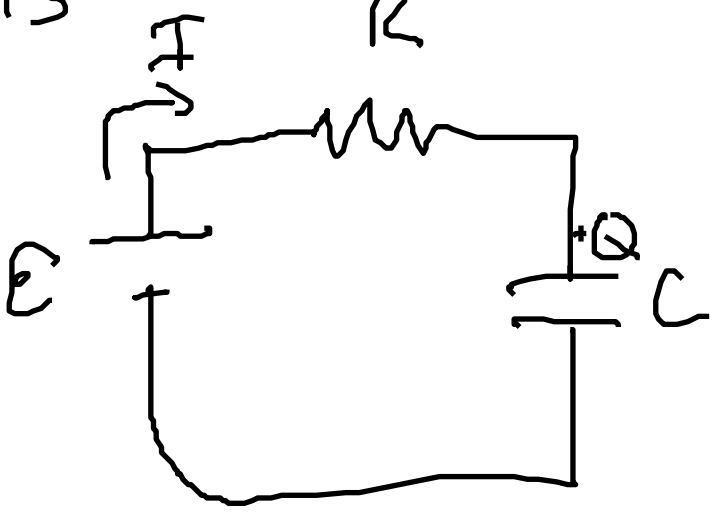
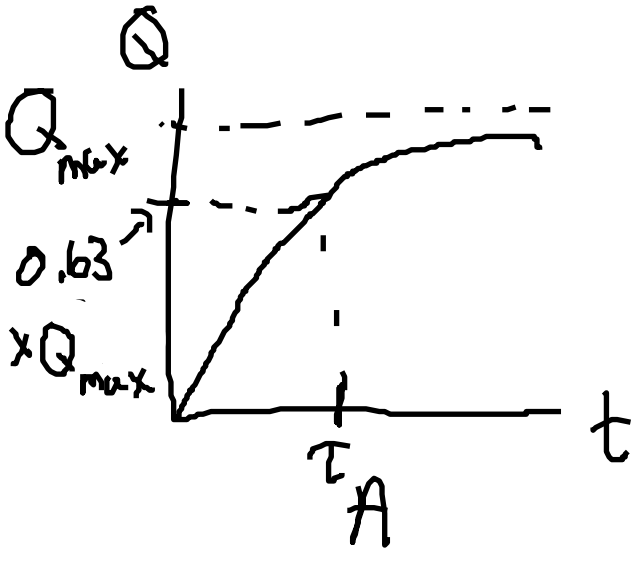
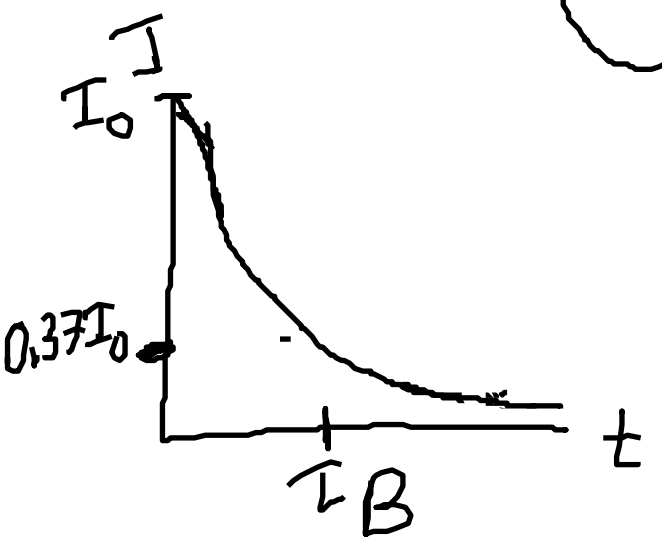
RC Circuits

- charging







$$Q(t) = Q_{max}\left(1 - e^{-t/RC}\right)$$

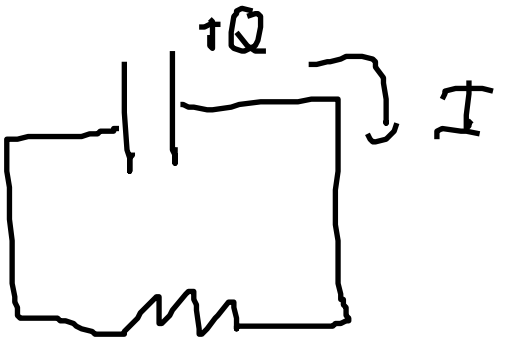
$$I(t) = I_0 e^{-t/RC}$$

$$Q_{max} = C\mathcal{E} \qquad I_0 = \frac{\mathcal{E}_0}{R}$$

$\tau = RC$: time constant

time needed for $e^{-t/\tau}$ to reach 37%

- discharging



If I is positive

Q is decreasing

$$I = -\frac{dQ}{dt}$$

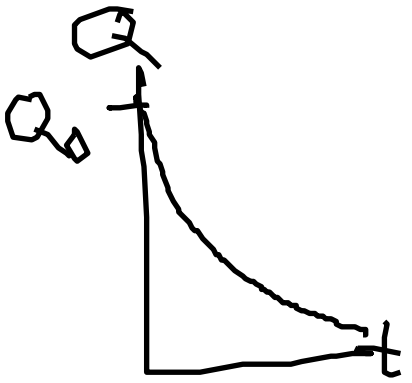
(sometimes t, sometimes $-$)

$$Q = Q_0 e^{-t/RC}$$

Q_0 = given initial charge

$$I = I_0 e^{-t/RC}$$

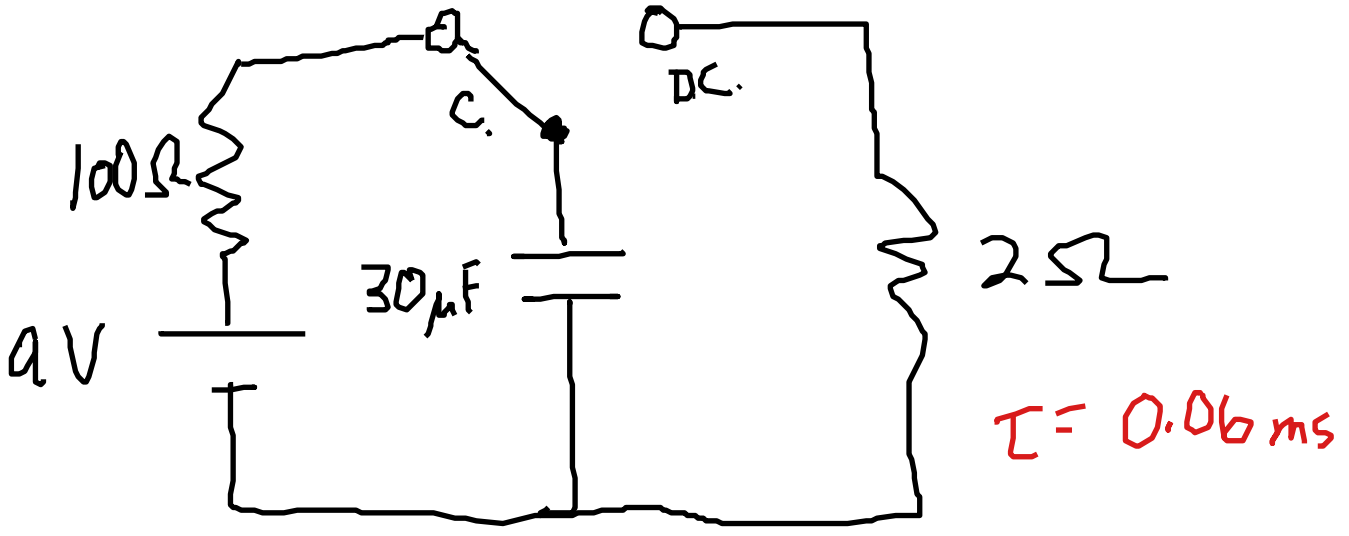
$$I_0 = \frac{\Delta V}{R} = \frac{Q_0}{RC}$$



Suppose $C = 300\mu F$ and $R = 200\Omega$

How long for Q to reach ~37% of its initial value?

$\tau = RC = (200)(300 \times 10^{-6}) = 60ms$



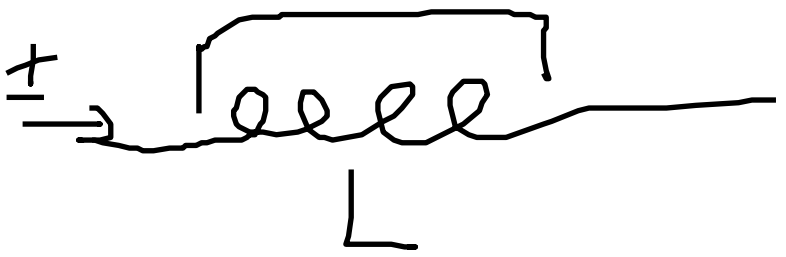
Will this capacitor charge or discharge faster?

Fast discharge, slow charge
- camera flash
- defibrillator

Fast charge, slow discharge
- electric car (theoretical)

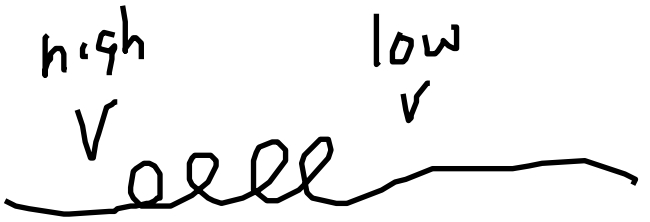
Inductors

$$\Delta V = -L \frac{dI}{dt}$$



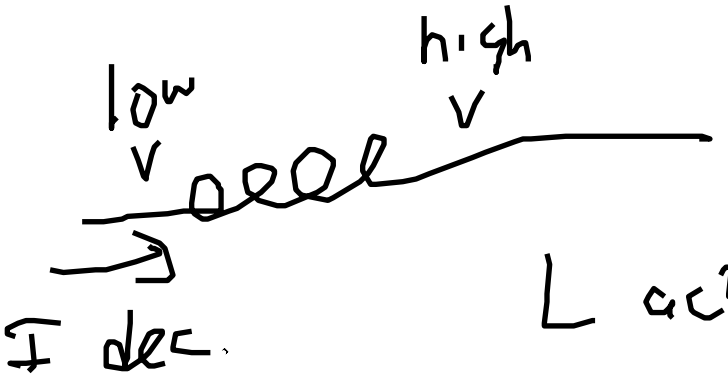
Inductance L: units of Henry

$$1\,H = 1\,\frac{Vs}{A} = 1\,\Omega s$$



$\rightarrow$
I inc.

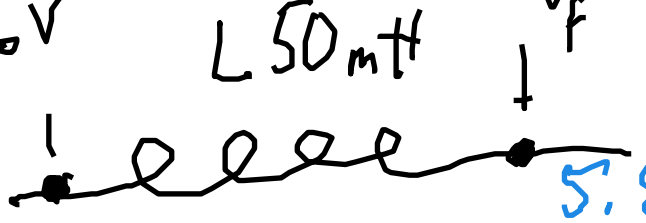
L acts like a resistor



I dec.

L acts like a battery

e.g.



$\vec{I}(t) = \left(4\,\frac{A}{s}\right)t$

increasing

$$\Delta V = -L \frac{dI}{dt}$$

$$= -(50 \times 10^{-3})(4)$$

$$= -200 \times 10^{-3} V$$

$$\Delta V = -0.2V$$

Alternating Current

$$V(t) = V_0 \cos(\omega t + \phi_0)$$

$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t + \phi)$$

$\mathcal{E}_0$: peak voltage

ω: angular frequency $\left(\frac{rad}{s}\right)$

$$\omega = 2\pi f$$

f: normal frequency (Hz)

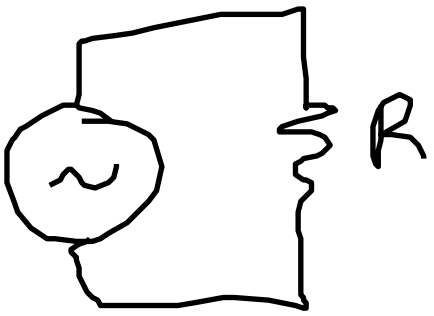
Current must also oscillate with ω

$$I(t) = I_0 \cos(\omega t + \phi_1)$$

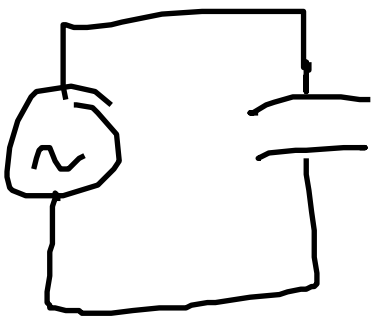
I_0: peak current

$$\mathcal{E}_0 = I_0 Z$$

a whole-circuit equation

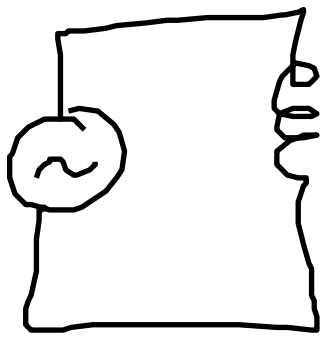


$$Z = R$$



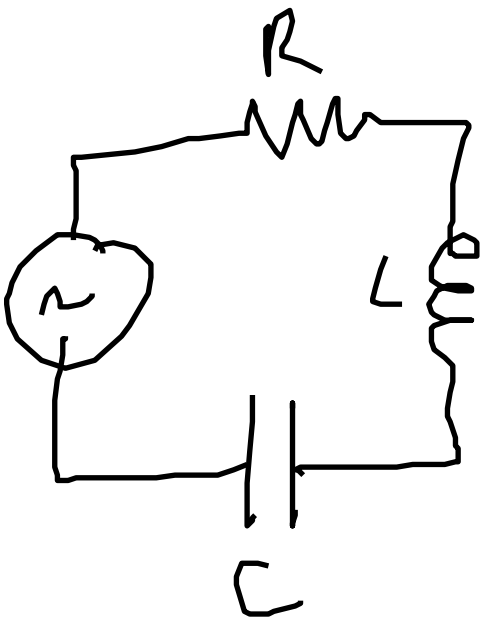
$$Z = \frac{1}{\omega C} = X_C$$

capacitive reactance



$$Z = \omega L = X_L$$

inductive reactance



$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$I(t) = I_0 \cos \omega t$$

$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t - \phi)$$

$$\tan \phi = \frac{X_C - X_L}{R}$$

$\phi > 0$: current leads voltage

$\phi < 0$: current lags voltage