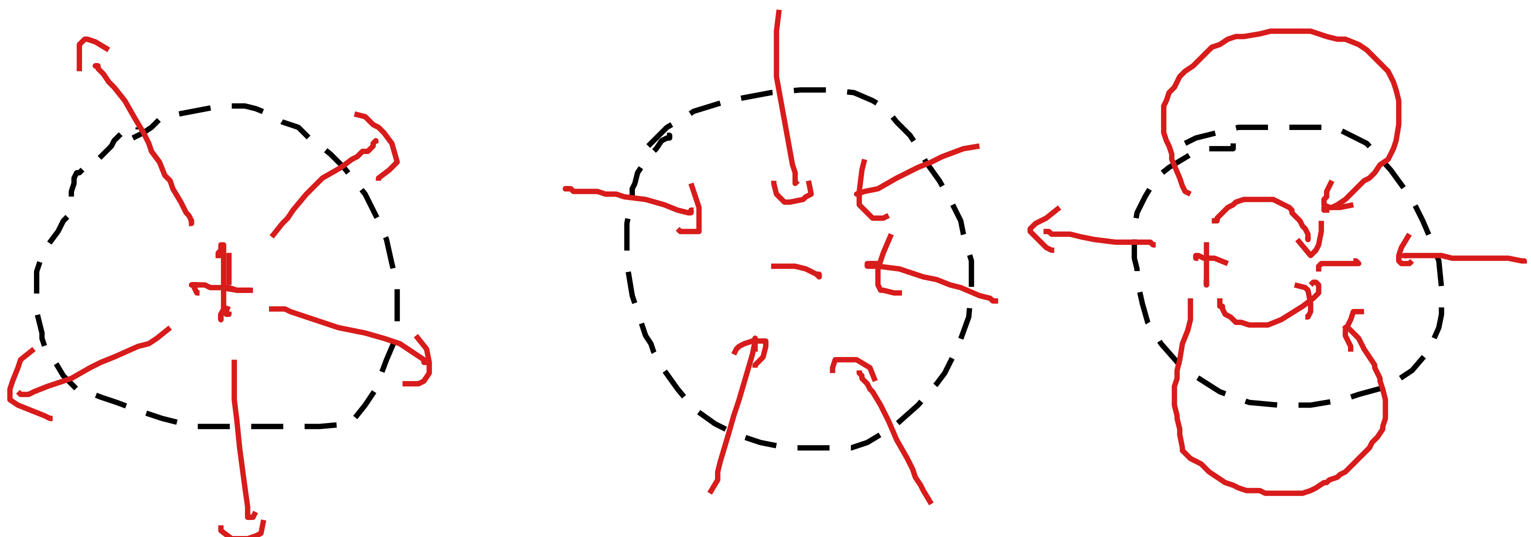
Maxwell's Equations

- Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$



positive $\int E \cdot dA$

negative $\int E \cdot dA$

zero $\int E \cdot dA$

- $$\oint \vec{B} \cdot d\vec{A} = 0$$

no magnetic monopoles

• Faraday's Law

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt}$$

potential difference

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

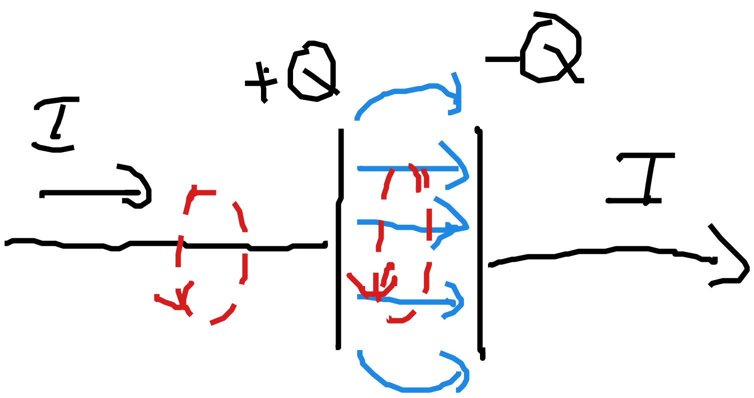
integral around closed loop

flux through that loop

- Ampère's Law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{thru} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell Law



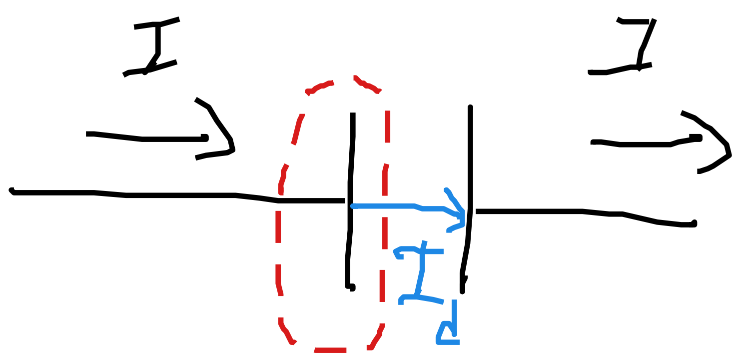
Φ_E increasing $\quad \frac{d\Phi_E}{dt}$

Same $\vec{B}$ inside as on wire even though no current

$$\varepsilon_0 \frac{d\Phi_E}{dt} = I_d$$ displacement current

I_d acts like a current in a couple ways

- creates a magnetic field
- makes conservation of current work



I flows in
I_d "flows" out

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 I_d$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \qquad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

"no "magnetic current""

$$\oint \vec{B}\cdot d\vec{A} = 0 \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Integral forms of Maxwell's Equations

↳ Derivative Forms

$$\nabla\cdot\vec{E} = \rho/\epsilon_0 \qquad \nabla\times\vec{E} = -\frac{d\vec{B}}{dt}$$

← charge density

current density

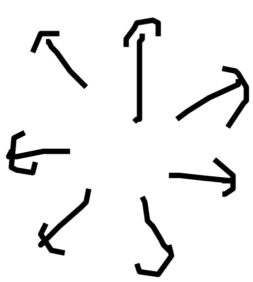
$$\nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{d\vec{E}}{dt}$$

divergences

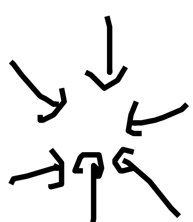
- scalars
- measure how much a field is spreading out

curls

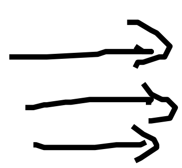
- vectors
- measure how much a field spins



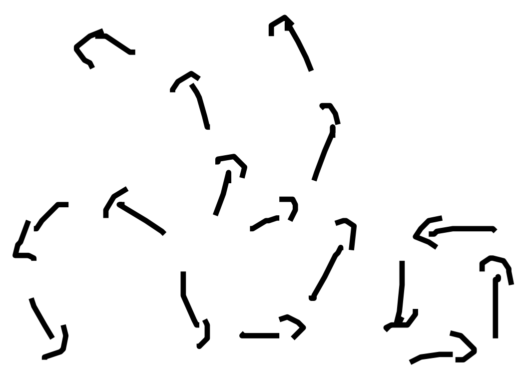
positive divergence

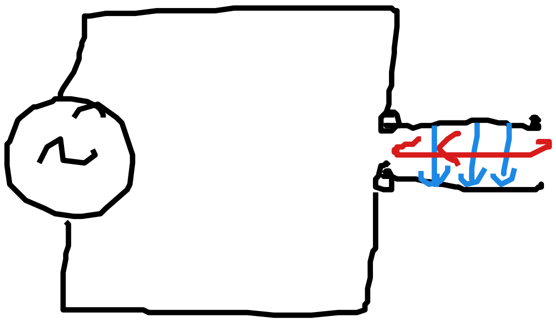


negative



zero





Alternating $\vec{E}$ in capacitor

creates an alternating $\vec{B}$

which creates an alternating $\vec{E}$

which . . .

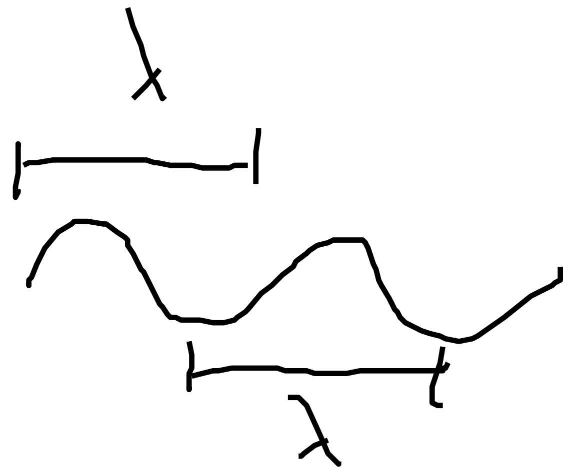
these fields propagate outward in space

this is a wave

Wave: a propogation of a disturbance in a specific direction

- speed v
- wavelength λ
- frequency f

  period T : time for cycle to reset



We can solve Maxwell's Equations with a wavelike solution with speed

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times 10^{8} \, m/s$$

$\mu_0 = 4\pi\times10^{-7}$ $\quad \epsilon_0 = 8.85\times10^{-12}$

Electromagnetic Waves
travel at speed of light

⇒ Light is an EM Wave