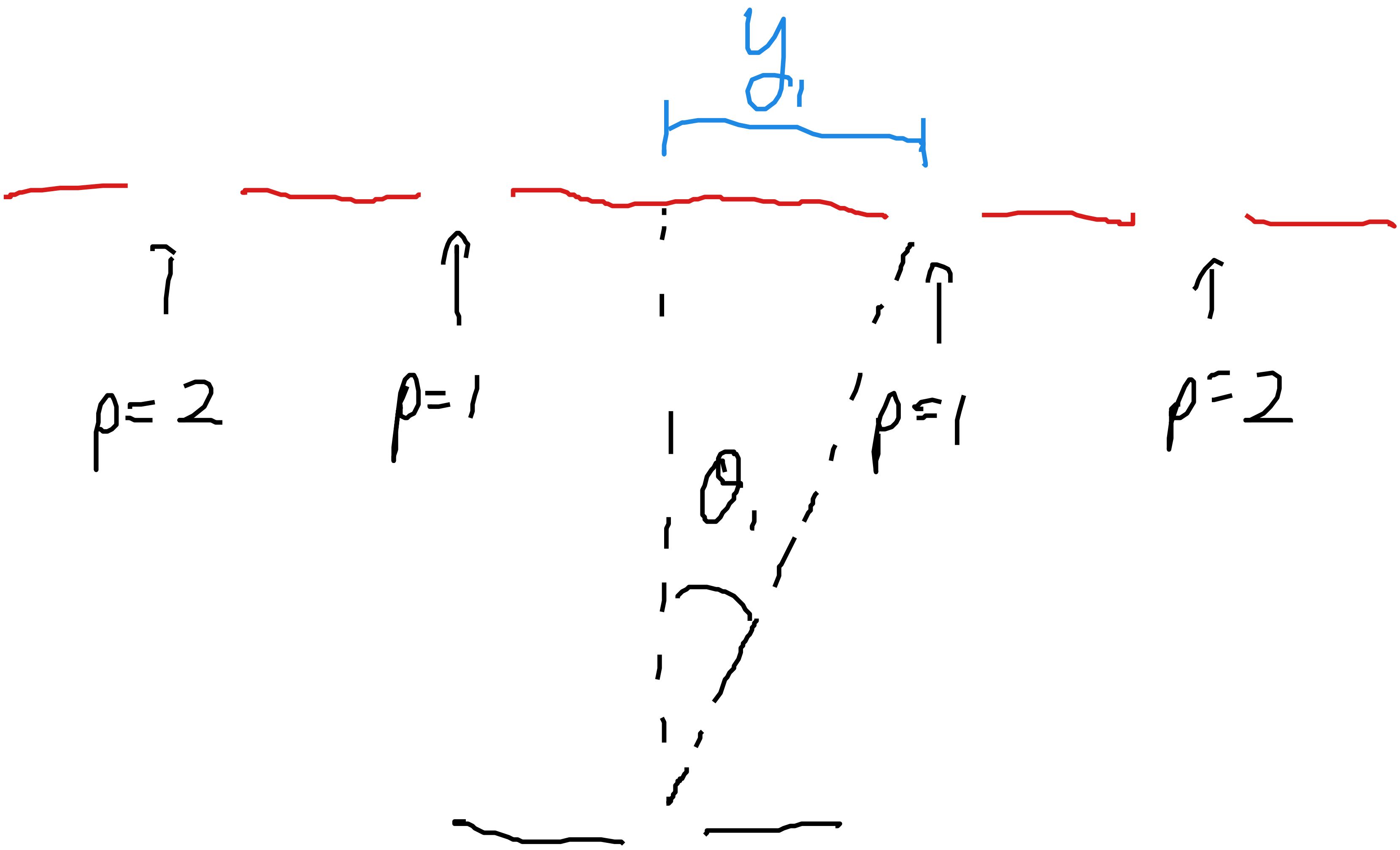
Diffraction Patterns



$$\boxed{\sin\theta_p = \frac{p\lambda}{a}}$$

For a single slit

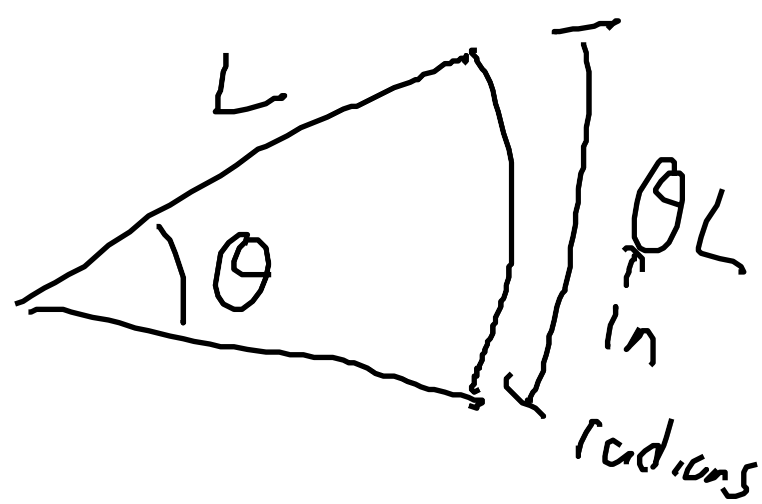
If $\theta_p \lesssim 20^\circ$

$$\sin\theta \simeq \tan\theta \simeq \theta$$ in radians

$$\theta_p \simeq \frac{p\lambda}{a}$$

in radians

$$y_p \simeq \frac{p\lambda}{a} L$$



For a circular opening with diameter a

depends on your calculator settings

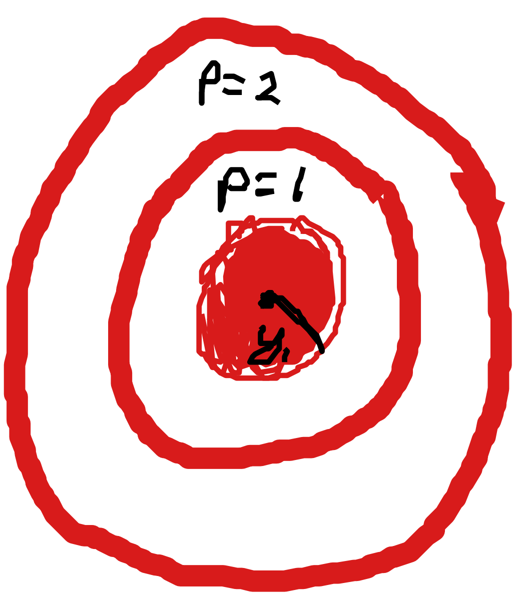
$$\sin\theta_p = 1.22\frac{p\lambda}{a}$$

radians

$\theta_p \lesssim 20^\circ$

$$\theta_p \approx 1.22\frac{p\lambda}{a}$$

$$y_p \approx 1.22\frac{p\lambda}{a}L$$



Central bright spot has radius

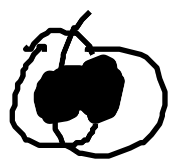
$$y_1 \approx 1.22\frac{\lambda L}{a}$$

diameter/width $= 2.44\frac{\lambda L}{a}$

This is minimum width of an object that you can perceive

 Reality

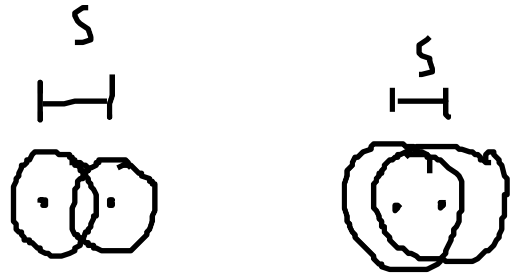
(diffraction pattern created by pupils)

 Seen

Small sources close together may appear to be a single source

Whether we can distinguish two objects is determined by

Rayleigh Criterion

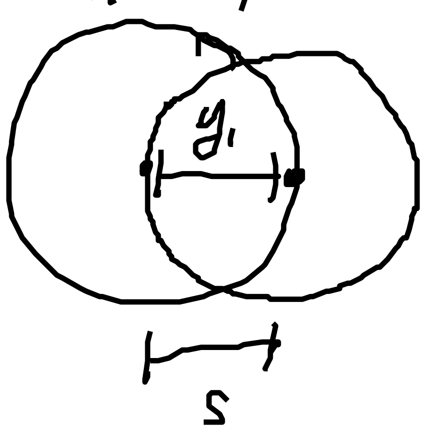


yes two sources

looks like 1 source

If center of one spot is overlapped by other bright spot, then they cannot be distinguished

limiting condition



To be distinguishable,

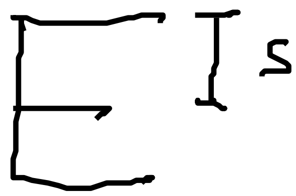
$$S \geq y_1$$

$$S \geq 1.22\,\frac{\lambda L}{a}$$

$$\sin\theta_s \geq 1.22\frac{\lambda}{a}$$

minimum requirement
for angular separation
to distinguish two objects

$$s \geq 1.22\frac{\lambda L}{a}$$



To read this E, we need the separation between branches to be

$$s \geq 1.22 \frac{\lambda L}{a}$$

pupil diameter

$$a = \begin{cases} 2\text{-}4\,\text{mm} & \text{light} \\ 4\text{-}8\,\text{mm} & \text{dark} \end{cases}$$

$$\lambda = 500\,\text{nm}$$

12 pt font   12 pts = 4 mm

$$s = 2\,\text{mm}$$

$$2\,mm \geq 1.22 \frac{(500\,nm)}{(4\,mm)} L$$

$$L \leq \frac{(2\,mm)(4\,mm)}{1.22(500\,nm)}$$

$$L \leq \frac{(2\times10^{-3})(4\times10^{-3})}{1.22(500\times10^{-9})}$$

$$L \leq 13.1\,m$$

absolute maximum distance you can read 12pt E.

In practice, probably a lot less

$$\left.\begin{aligned} S_{min} &= 1.22\frac{\lambda L}{a} \\ \theta_{min} &= 1.22\frac{\lambda}{a} \end{aligned}\right] \text{maximum resolution}$$

Resolution improves when S_{min} or θ_{min} is smaller

- bigger aperture
- smaller λ (purple, UV, Xrays)

In astronomy, "distances" in the sky are usually measured in angles

Moon's width: $\frac{1}{2}^\circ \sim 0.09$ rad

Sun's width: $\frac{1}{2}^\circ$ (same!)

arm's length

stars

$\lambda = 500\,nm$

$a = 6\,mm$

(dark)

What is the minumum angle θ so that you can distinguish these two stars?

$$\theta_{min} = 1.22\frac{\lambda}{a}$$

$$= 1.22\frac{(500\times10^{-9})}{(6\times10^{-3})}$$

$$= 0.0001 \text{ radians}$$

$$\times\frac{180^\circ}{\pi \text{ rad}} = 0.006^\circ$$