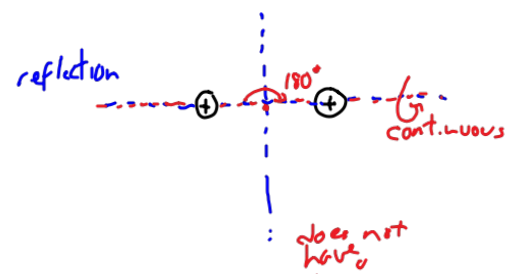
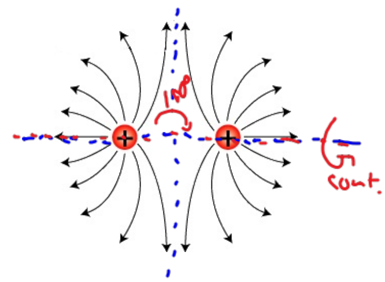
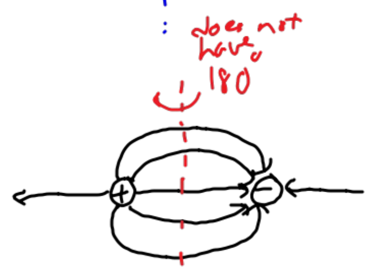
☆☆ If a set of source charges has a certain symmetry, their electric field has the same symmetry, & vice versa.







We can use Symmetry & Gauss' Law to find the electric field of

- sphere
- infinite cylinder
- infinite plane

Sphere

- continuous rotational sym. around any axis through its center
- reflection sym. across any mirror through its center

"spherical symmetry": possessing those two sets of symmetries

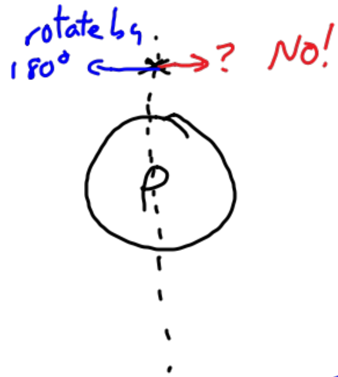
e.g. solid sphere
spherical shell
concentric combinations of above 
~~point~~ •

"Spherical Symmetry"

- has all the symmetries of a sphere
- solid sphere, spherical shell, point

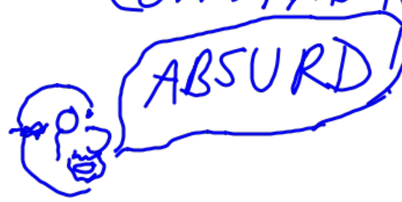
Any charge distribution with spherical symmetry has an electric field with S.S. as well.

solid sphere:

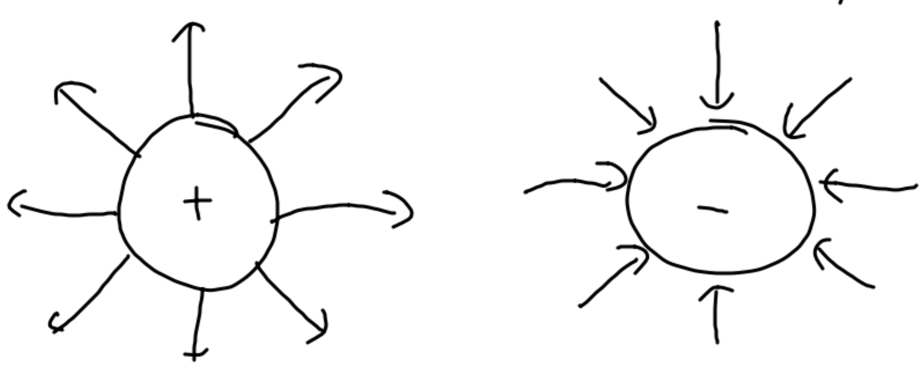


If field points right, it must also point left.

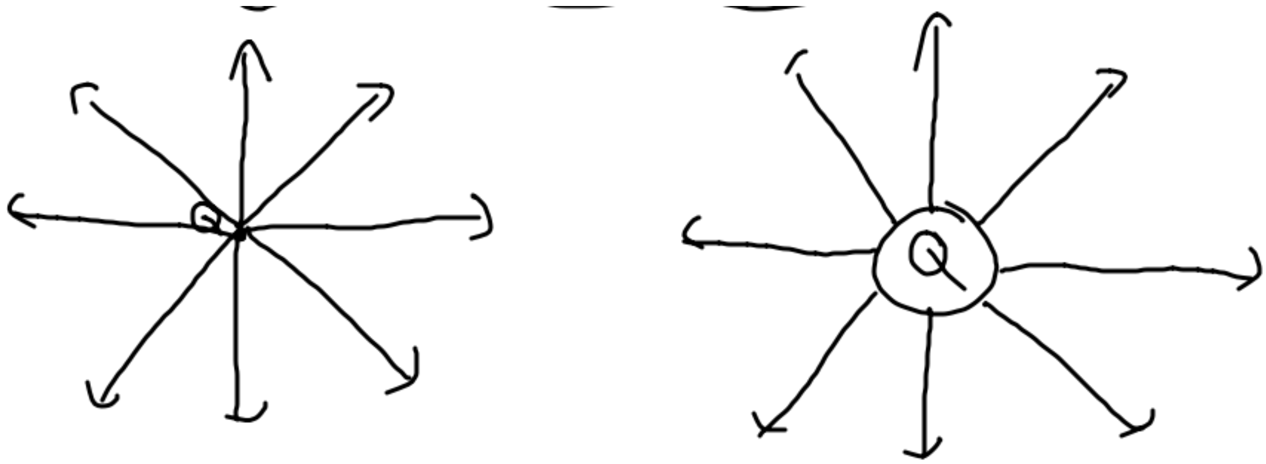
CONTRADICTION!



Electric field lines will either ALL point away from center or towards the center of the sphere.

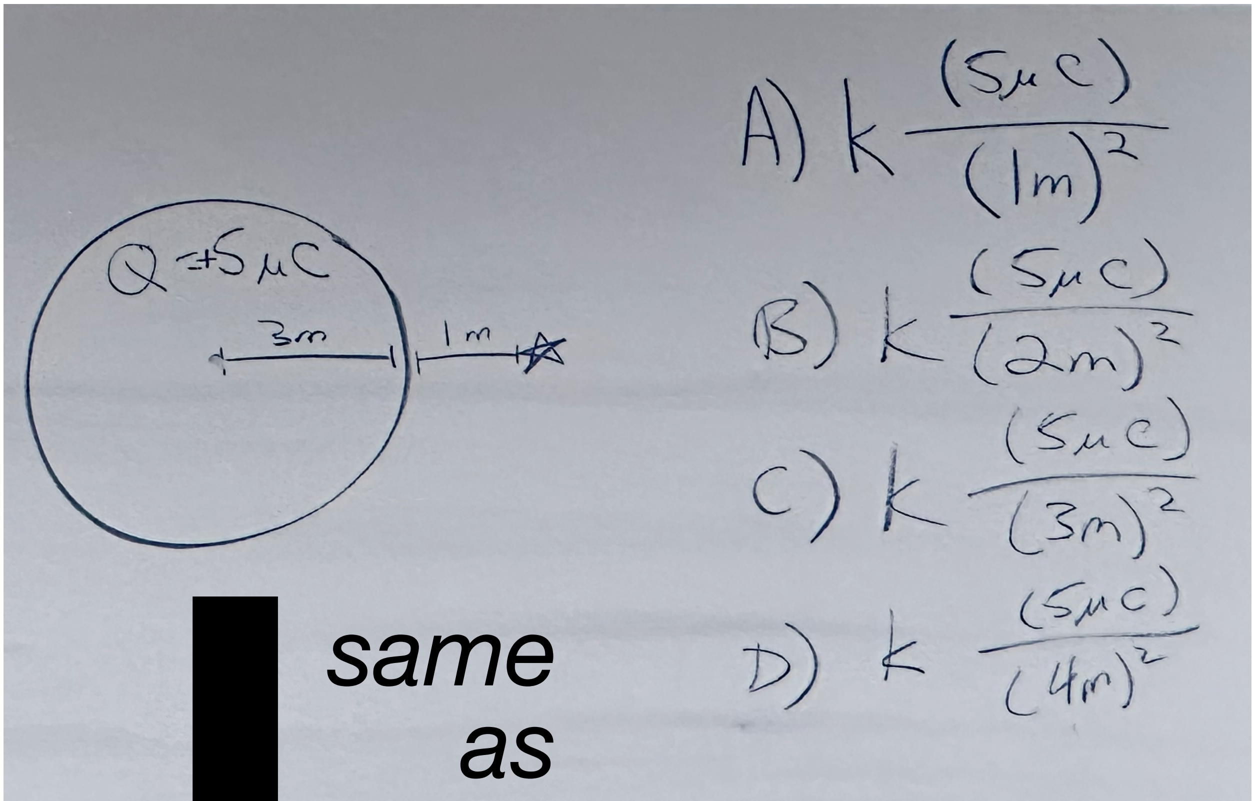


$|\vec{E}|$ only depends on distance from center of sphere, not "latitude or longitude."

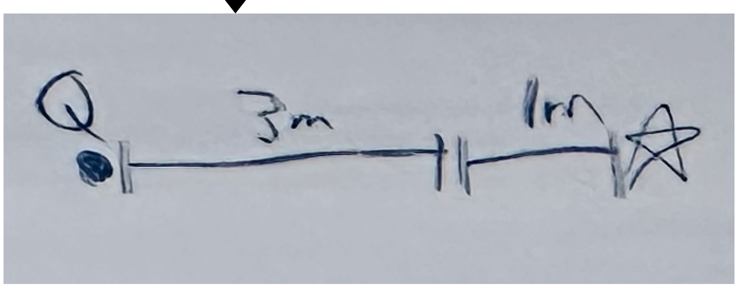


- field far away from both point & sphere is same: $\vec{E} = k \frac{Q}{d^3} \vec{d}$
  ($\vec{d}$ points away from center)
- denser the field lines, stronger the field
- these field lines converge in exactly the same way, & so $\vec{E}$ grows in exactly the same way as you approach either point or sphere.

$\therefore$ Electric field outside a system with spherical symmetry is same as field of a point with same total charge Q.



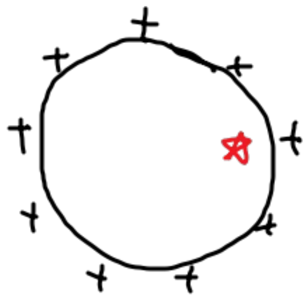
same as



D is the correct answer

Inside a Spherical Shell of Charge

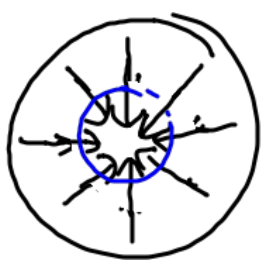
$\sigma > 0$



$\vec{E}$ at ☆ is

A) $\leftarrow$ B) 0 C) $\rightarrow$

Suppose $\vec{E}$ points $\leftarrow$



Φ through blue sphere is negative.

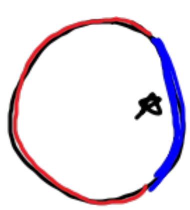
$\therefore$ must be a negative charge inside.

But there isn't.

$\therefore$ $\vec{E}$ does not point to left at the star.

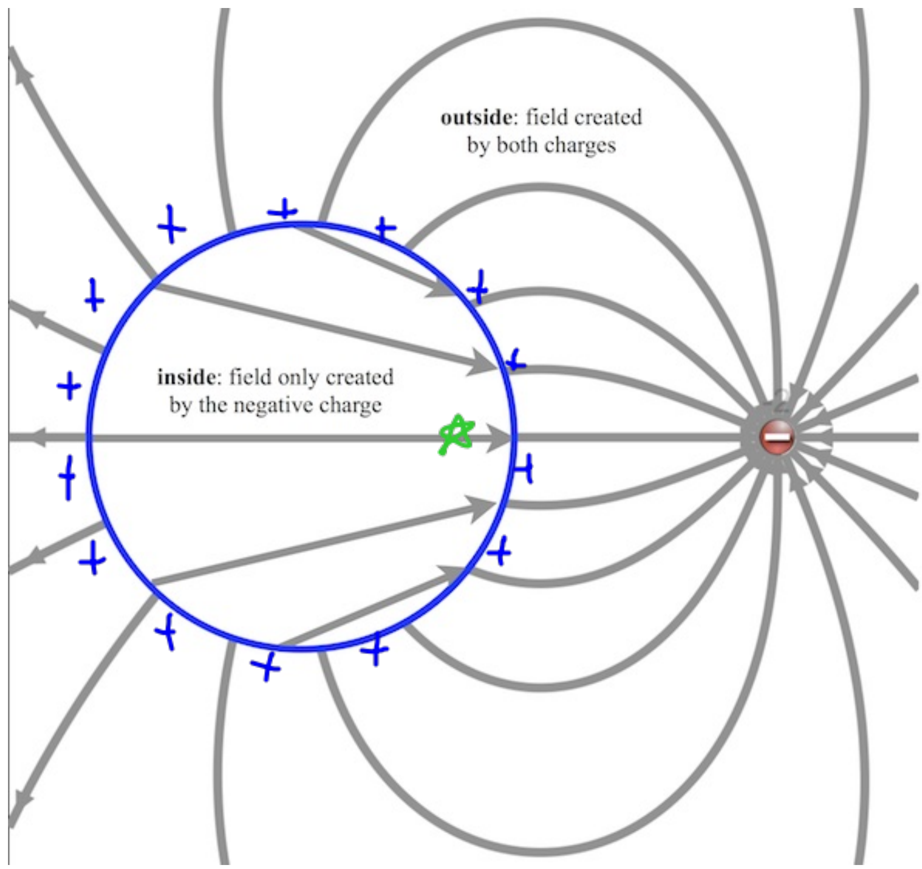
Electric Field at every point inside a spherical shell is zero.

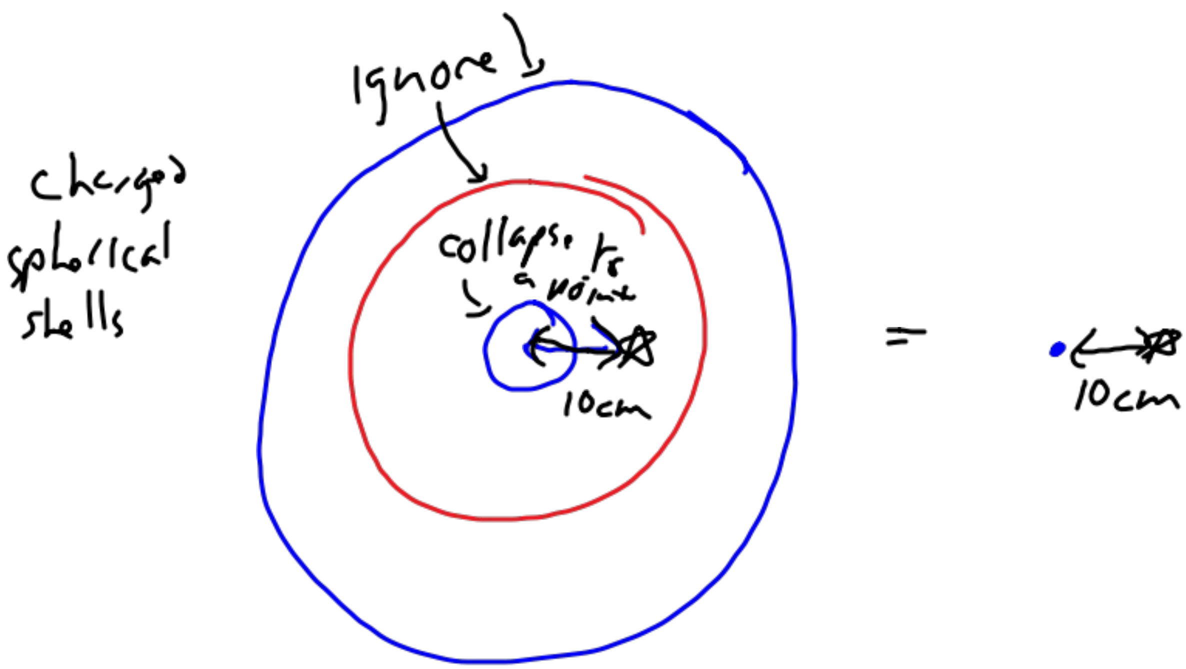
Charges are farther but more numerous, $\vec{E} \rightarrow$



Charges are closer create field to the left

Revised statement: A charged spherical shell creates no electric field inside itself.





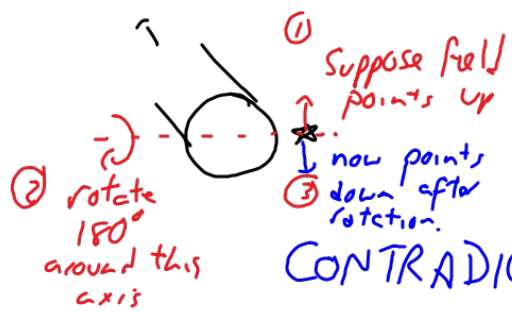
Infinite Cylinder

Why? Good approximation
for long cylinders or
close to any cylinder

Symmetries
- continuous rotational S. around Axis
- 180° rotation, any axis ⊥ to Axis
- reflection across mirror ⊥ to Axis
  " " " containing Axis
- translation along its axis

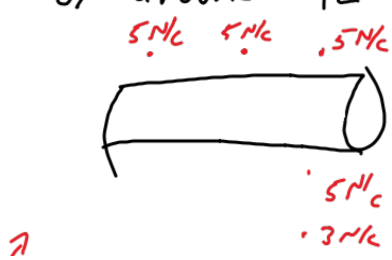
- ∞ cylindrical shell
- ∞ solid cylinder
- ∞ line

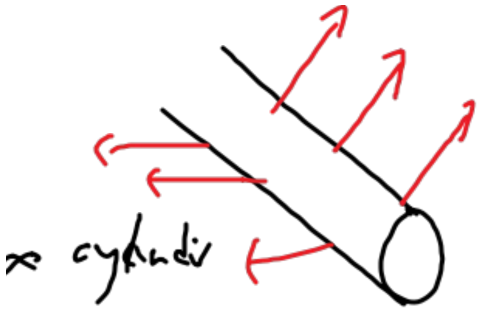
What could $\vec{E}$ look like?

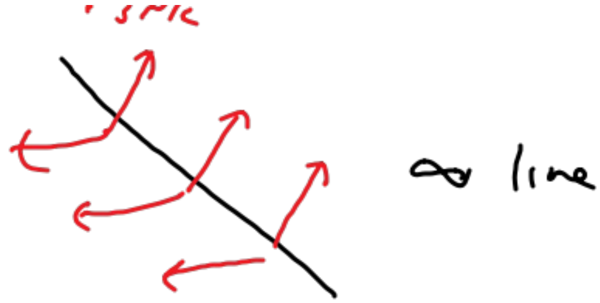


CONTRADICTION!

- $\vec{E}$ points away from or towards Axis
- $|\vec{E}|$ only depends on distance from Axis, not position along the Axis or around the cylinder.

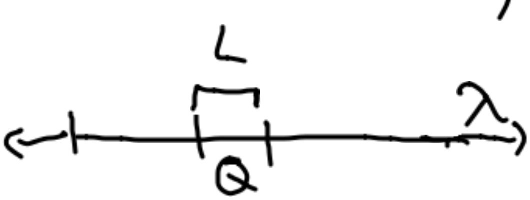






- identical field lines outside cylinder & away from ∞ line
- far away from ∞ cylinder, has same field as ∞ line

$\therefore$ $\vec{E}$ outside any system with ∞ cylindrical symmetry is same as if you collapsed that system to an ∞ line with the same linear charge density λ.



$$Q = \lambda L$$

$$Q = \rho V = \rho \pi R^2 L$$

$$\text{“}\lambda\text{”} = \frac{Q}{L} = \rho \pi R^2$$