$$\vec{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

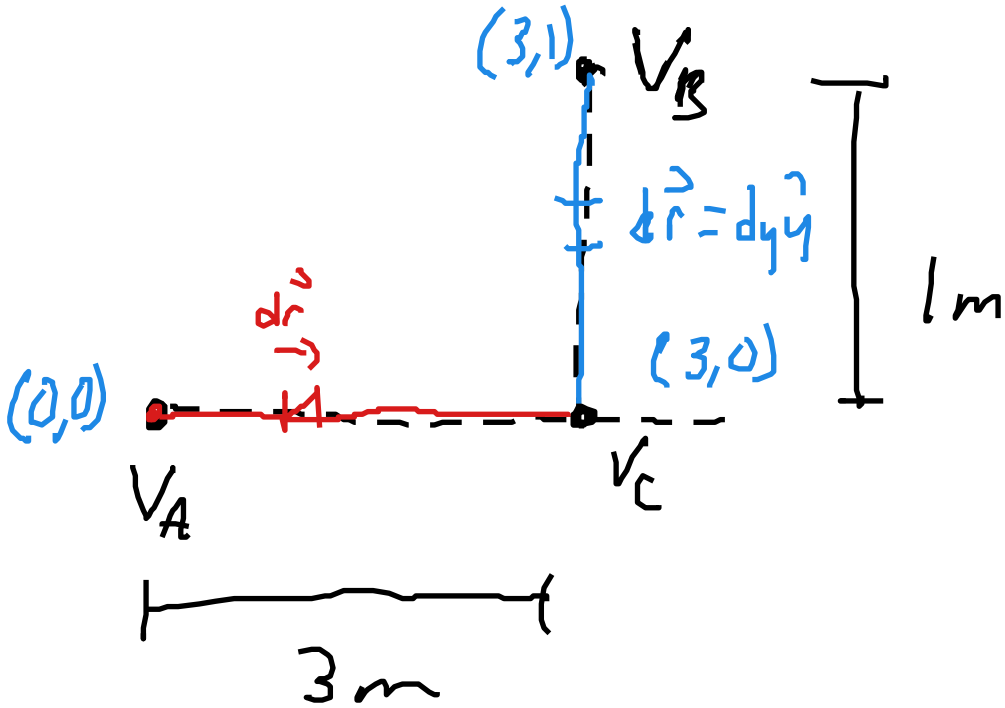
e.g $\quad V = xy + z$

$$E_x = -\frac{\partial}{\partial x}(xy+z) = -[y+0]$$

$$= -y$$

$$E_z = -1$$

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{r}$$



$$V_C - V_A = -\int \vec{E} \cdot (dx\,\hat{x})$$

$$= -\int_0^3 (\vec{E} \cdot \hat{x})\, dx$$

$$= -\int_0^3 E_x(y=0)\, dx$$

$$V_B - V_C = -\int \vec{E} \cdot (dy\,\hat{y})$$

$$= -\int_0^1 E_y(x=3)\,dy$$

Suppose $\vec{E} = (x+y)\,\hat{x} + (x-y)\,\hat{y}$

$$V_C - V_A = -\int_0^3 x + \overset{0}{\overset{\downarrow}{y}}\;dx$$

$$= -\int_0^3 x\,dx = -\left[\frac{1}{2}x^2\right]_0^3$$

$$= -\frac{1}{2}(9) = -\frac{9}{2}$$

$$V_B - V_C = -\int_0^1 (\overset{3}{\overset{\downarrow}{x}} - y)\,dy$$

$$= -\int_0^1 3 - y\;dy$$

$$= -\left[3y - \frac{1}{2}y^2\right]_0^1 = -\left[3 - \frac{1}{2}\right]$$

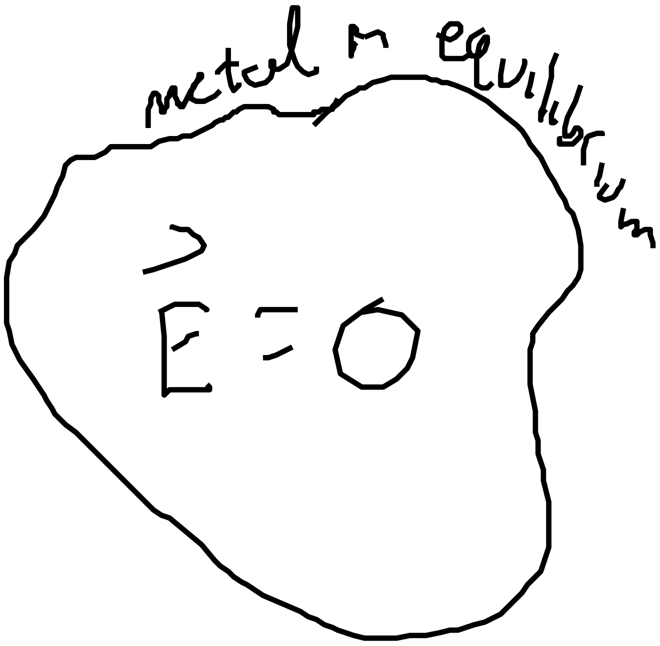
$$= -\frac{5}{2}$$

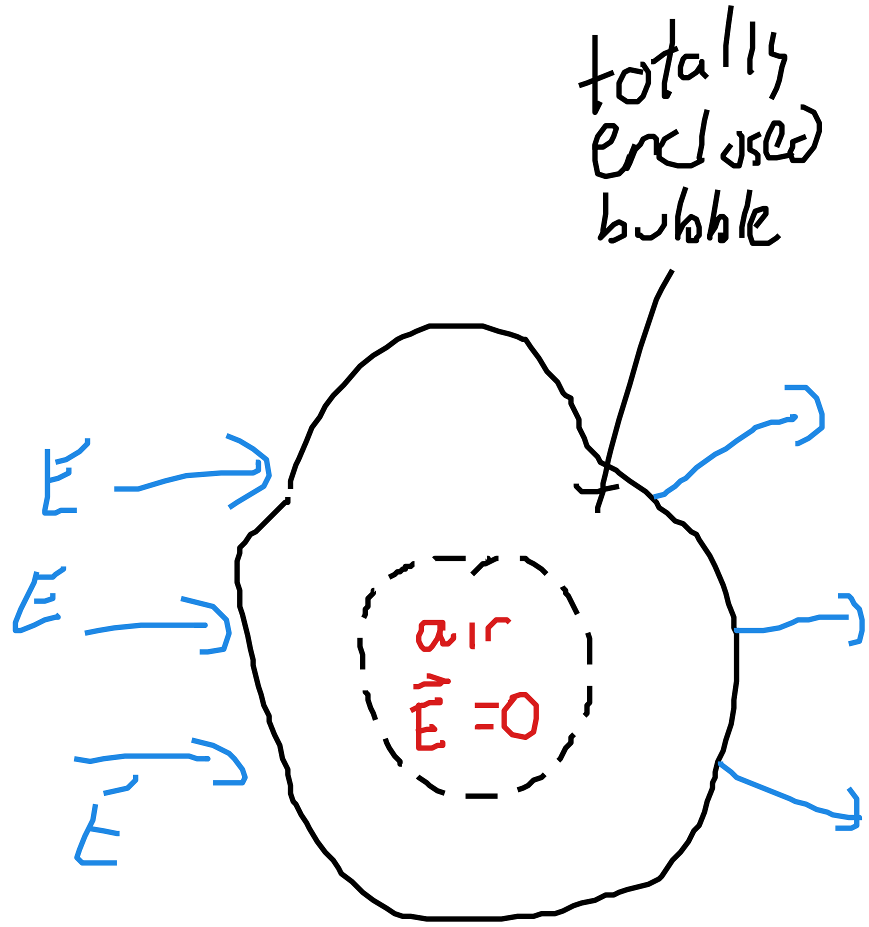
$$\cancel{V_C} - V_A = -\frac{9}{2}$$

$$+ \quad V_B - \cancel{V_C} = -\frac{5}{2}$$

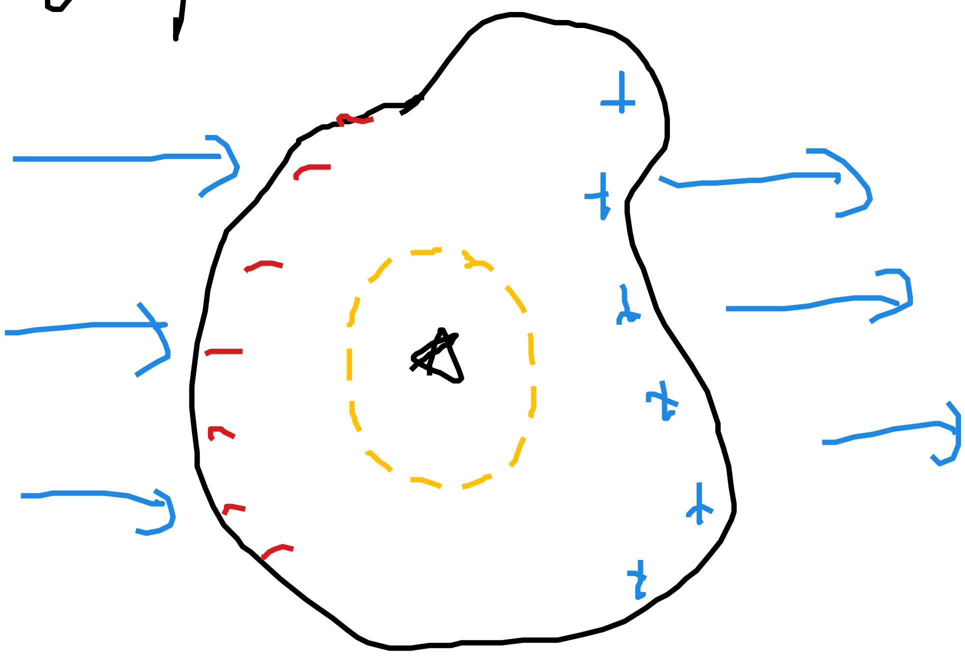
$$V_B - V_A = -7$$

Conductors





Why?

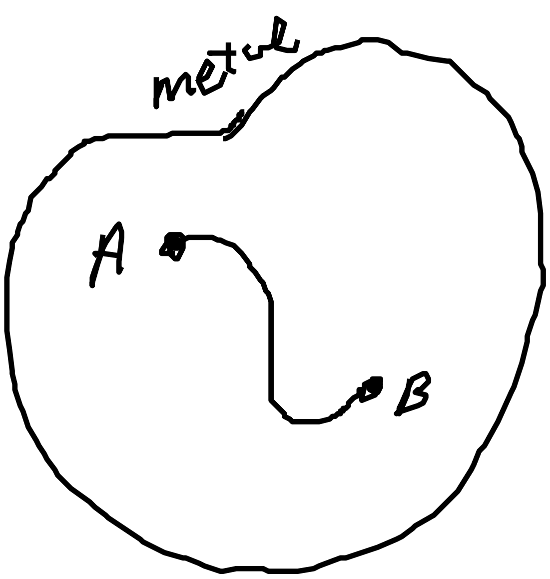


Before cavity forms, $\vec{E} = 0$ at ☆

If I make the cavity appear, charges don't change so $\vec{E}$ doesn't change → remains zero

Faraday cage

- RFID - blocking foil-lined wallets
- window of microwave
- cars are partial protection for lightning
- USB and coaxial cables



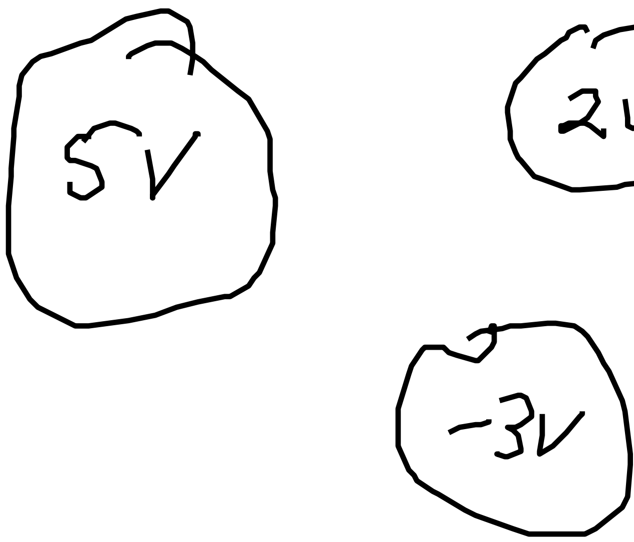
$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$= -\int 0 \, d\vec{r}$$

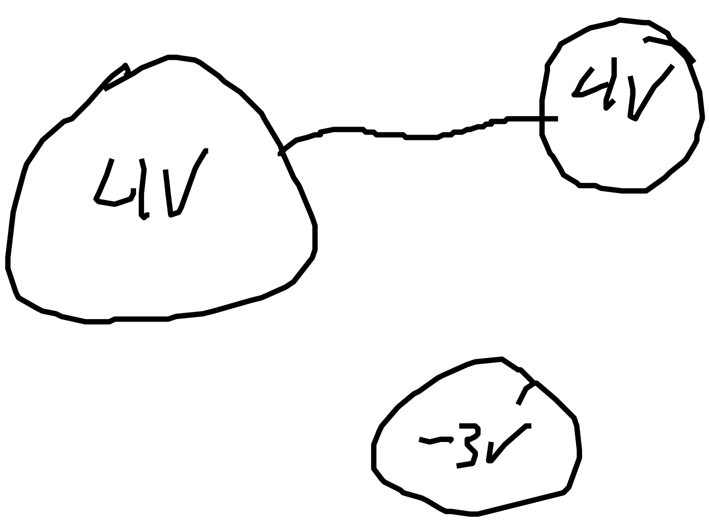
$$= 0$$

All points in or on a conductor (in equilibrium) have the same potential

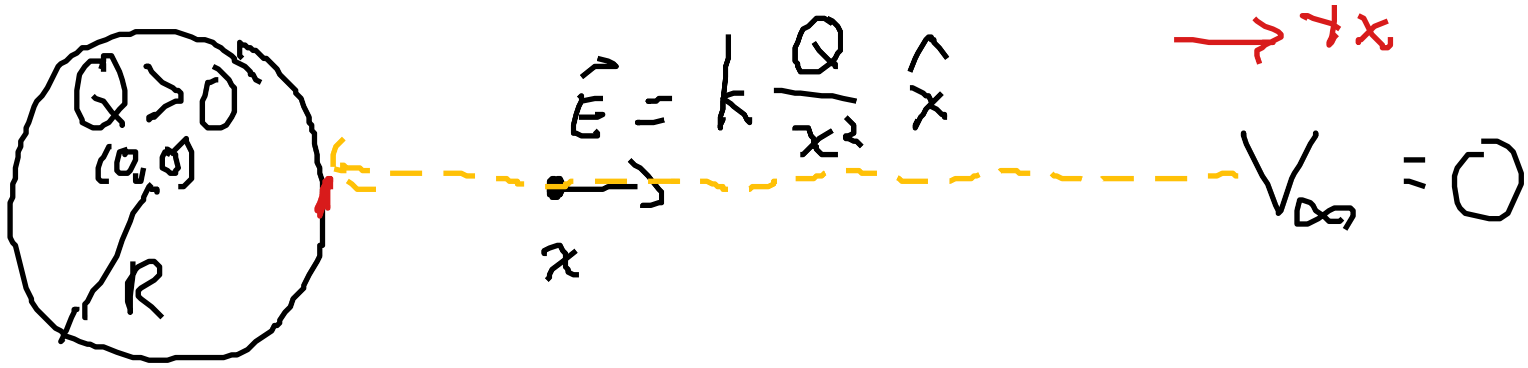
Different conductors can have different potentials though



Connect two pieces of metal with a wire, or solder, etc that turns them into a single piece of metal.



Spherical conductor in equilibrium



$$\vec{E} = k\frac{Q}{x^2}\hat{x}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r} \qquad \vec{E}\cdot\hat{x} = E_x$$

$$V(R) - V(\infty) = -\int_\infty^R \vec{E}\cdot(dx\,\hat{x})$$

corrected

$$= -\int_\infty^R E_x\,dx$$

$$= -\int_\infty^R k\frac{Q}{x^2}\,dx$$

$$= -kQ\int_\infty^R x^{-2}\,dx$$

$$= -kQ\left[-x^{-1}\right]_\infty^R = -kQ\left[-\frac{1}{x}\right]_\infty^R$$

$$= kQ\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$= \frac{kQ}{R}$$

not $-\hat{x}\,dx$

My mistake from class:
we should write $d\vec{r} = \hat{x}\,dx$

so $V(R) - V_\infty = -\int_\infty^R E_x\,dx$

and that eliminates the minus sign.

I have fixed the notes to be correct.