$$Q = C\Delta V \qquad C = \frac{Q}{\Delta V}$$

$$PE = \frac{1}{2}\frac{Q^2}{C}$$

$$PE = \frac{1}{2}C(\Delta V)^2$$

$$PE = \frac{1}{2}\left(\frac{Q}{\Delta V}\right)(\Delta V)^2$$

$$= \frac{1}{2}Q\Delta V$$

Batteries vs capacitors

Batteries	Capacitors
• store energy chemically	• store energy electrically
• some loss during conversion	• less loss of energy
• rather slow	• rather fast
• fail over time	• more robust
• high energy density	low energy density :(

Sphere with radius $R = 1m$

$$C = \frac{R}{k} = \frac{1m}{9\times10^{9}\ Nm^2/C^2}$$

$$= 1.1\times10^{-10}\ F$$

$$PE = \frac{1}{2}C(\Delta V)^2$$

standard high voltage in houses (stove dryer)

$$= \frac{1}{2}(1.1\times10^{-10}F)(220V)^2$$

$$= 2.66\times10^{-6}\ J$$

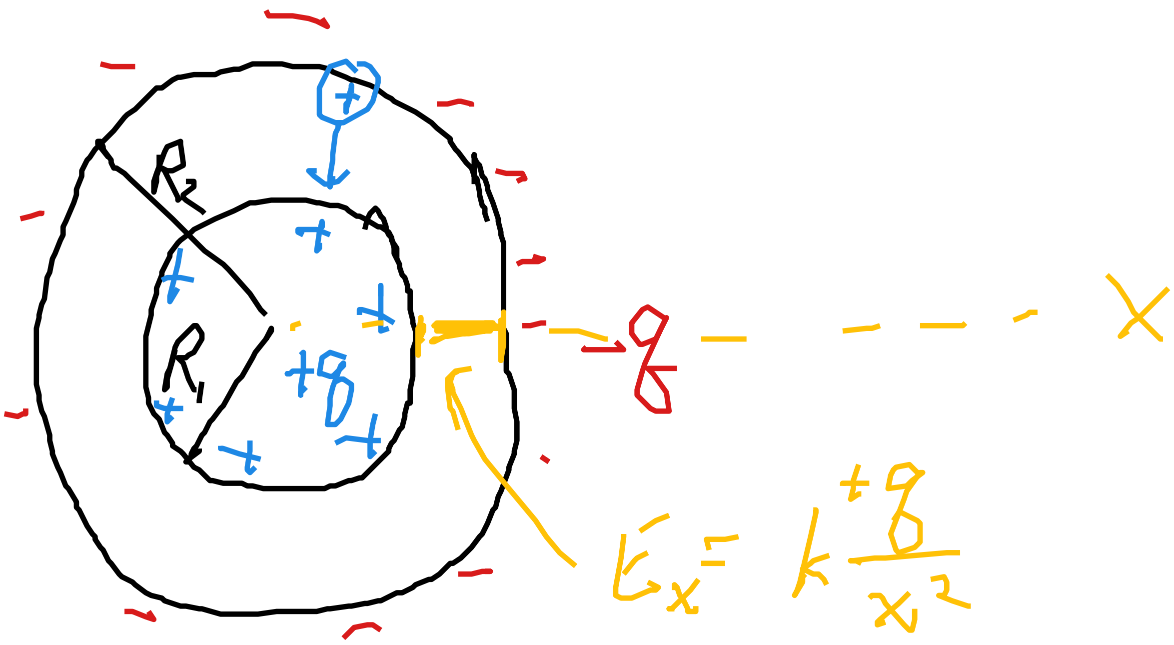
$W = \frac{J}{s}$

Phone might use 100mW

$$\frac{2.66\times10^{-6}\ J}{100\times10^{-3}\ J/s} = 26\mu s.$$

How to increase capacitance without increasing size?

Move "ground" closer



$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$= -\int E_x\, dx$$

$$= -\int_{R_2}^{R_1} k\frac{q}{x^2}\, dx$$

$$= -kq\left[-\frac{1}{x}\right]_{R_2}^{R_1}$$

$$= kq\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Delta V = kq\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

$$C = \frac{q}{\Delta V} = \frac{\cancel{q}}{k\cancel{q}\,\frac{R_2 - R_1}{R_1 R_2}}$$

$$= \frac{1}{k}\,\frac{R_1 R_2}{R_2 - R_1} = \frac{1}{k}\,\frac{R_1 R_2}{\Delta R}$$

gap between spheres

Make gap very very small

and C gets very very big

Two-piece capacitors

Problem: gap can't be

smaller than a few

atoms or else the +q

and -q will discharge.

$$C = \frac{1}{k}\frac{R_1 R_2}{\Delta R}$$

if $R_2 - R_1$ is tiny

$$C \simeq \frac{1}{k}\frac{R_1^2}{\Delta R}$$

best we can do

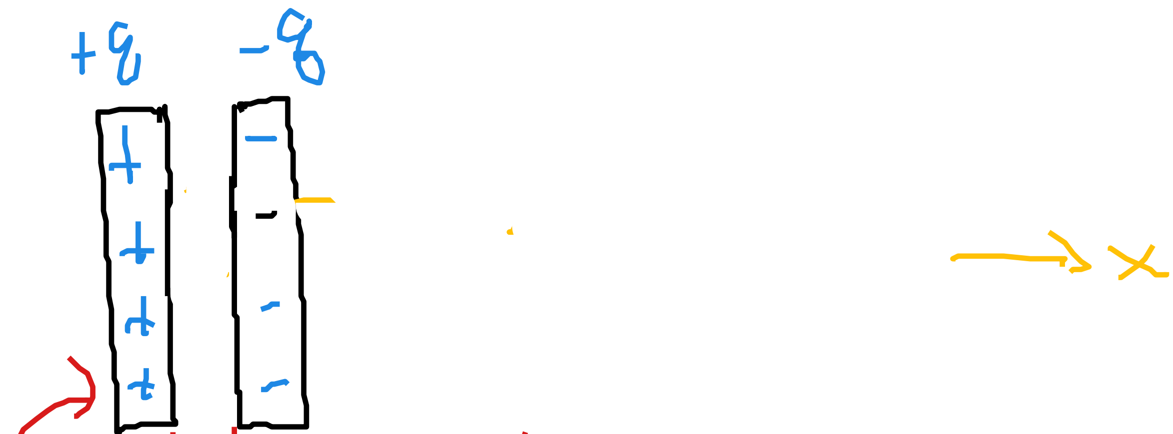
$$C = \frac{1}{(9\times10^9)}\frac{R_1^2}{(10^{-9}m)}$$

a few atoms

$$C \propto R^2$$

1m sphere : $C \sim 1F$

Parallel-Plate Capacitors



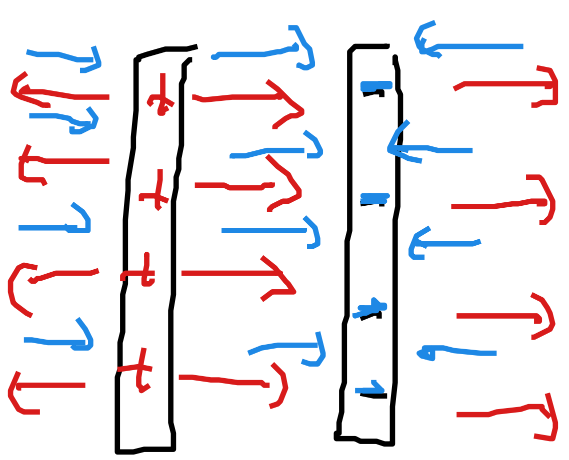
d: gap distance

A: area of each sheet

$$C = \frac{q}{\Delta V}$$

$$\Delta V = -\int E_x \, dx$$

Infinite sheets

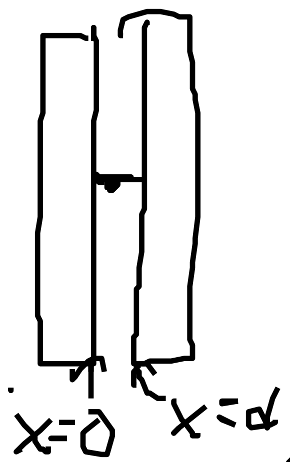


$+2\pi k\sigma$

$2\pi k\sigma$

E: 0 $4\pi k\sigma$ 0

At the center of two finite sheets, close together, field is also roughly $4\pi k\sigma$



$$\Delta V = -\int_d^0 4\pi k\sigma \, dx$$

$$= -4\pi k\sigma \int_d^0 dx$$

$$= -4\pi k\sigma \left[x \right]_d^0$$

$$= -4\pi k\sigma \left[0 - d \right] = 4\pi k\sigma d$$

$$C = \frac{q}{dV} = \frac{q}{4\pi k \sigma d}$$

$$\sigma = \frac{q}{A}$$

$$C = \frac{\cancel{q}}{4\pi k (\cancel{q}/A) d}$$

$$= \boxed{\frac{1}{4\pi k}} \frac{A}{d}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$$

$$\boxed{C = \epsilon_0 \frac{A}{d}}$$

parallel-plate capacitors