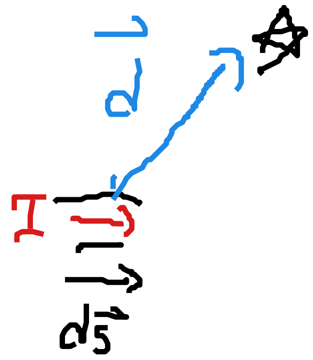
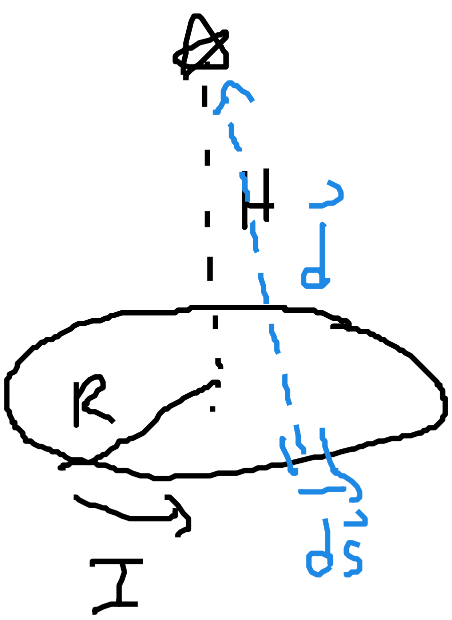


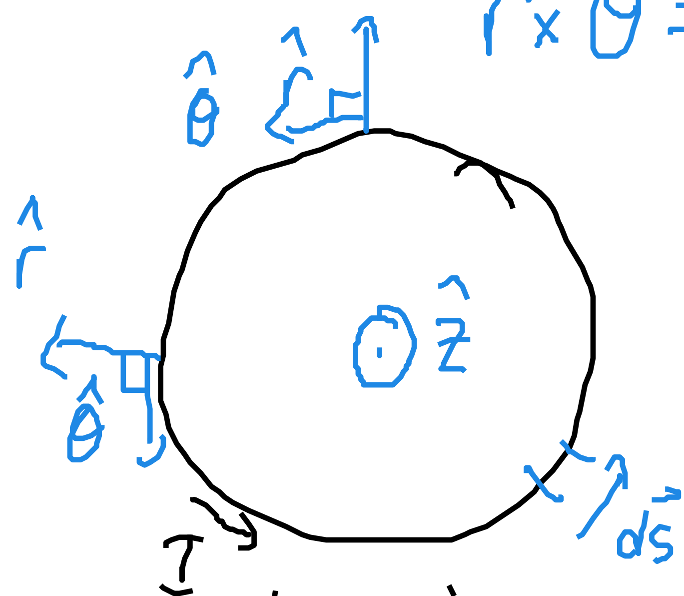
$$d\vec{B} = \frac{\mu_0}{4\pi} I \, \frac{d\vec{s} \times \vec{d}}{d^3}$$

$$\vec{B} = \int_{\text{segments}} d\vec{B}$$

$\hat{r} \times \hat{\theta} = \hat{z}$



side view



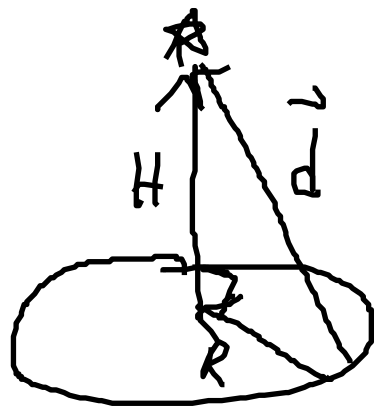
top view

not constants of integration $\left[\begin{array}{l} \hat{r} : \text{away from center} \\ \hat{\theta} : \text{tangential to circle} \end{array}\right.$

is constant $\rightarrow$ $\hat{z}$: along axis

$$d\vec{s} = \hat{\theta} R \, d\theta$$

$$\vec{d} = -R\hat{r} + H\hat{z}$$



$$d\vec{s} \times \vec{d} =$$

$$R\,d\theta\,\hat{\theta} \times \left(-R\hat{r} + H\hat{z}\right)$$

$$-R^2 d\theta \left(\hat{\theta} \times \hat{r}\right) + RH\,d\theta \left(\hat{\theta} \times \hat{z}\right)$$

$$-R^2 d\theta \left(-\hat{z}\right) + RH\,d\theta \left(\hat{r}\right)$$

$$|\vec{d}| = d = \sqrt{R^2 + H^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{s} \times \vec{d}}{d^3}$$

$$= \frac{\mu_0}{4\pi} I \left[\int_0^{2\pi} \frac{R^2 \hat{z}}{(R^2+H^2)^{3/2}} d\theta + \int_0^{2\pi} \frac{RH\hat{r}}{(R^2+H^2)^{3/2}} d\theta \right]$$

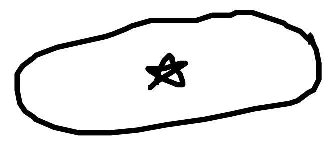
$$= \frac{\mu_0 I}{4\pi (R^2+H^2)^{3/2}} \left[R^2 \int_0^{2\pi} \hat{z}\, d\theta + RH \underbrace{\int_0^{2\pi} \hat{r}\, d\theta}_{\sum = 0} \right]$$

$$= \quad '' \quad \left[R^2 \hat{z} \int_0^{2\pi} d\theta + R \cdot 0 \right]$$

$$= \quad '' \quad \left[2\pi R^2 \hat{z} + 0 \right]$$

$$= \frac{\mu_0 I \cancel{2\pi} R^2 \hat{z}}{\cancel{4\pi}_{\,2} (R^2+H^2)^{3/2}} = \boxed{\frac{\mu_0 I R^2 \hat{z}}{2(R^2+H^2)^{3/2}} \quad \text{field along axis of a circular loop}}$$

$H = 0$



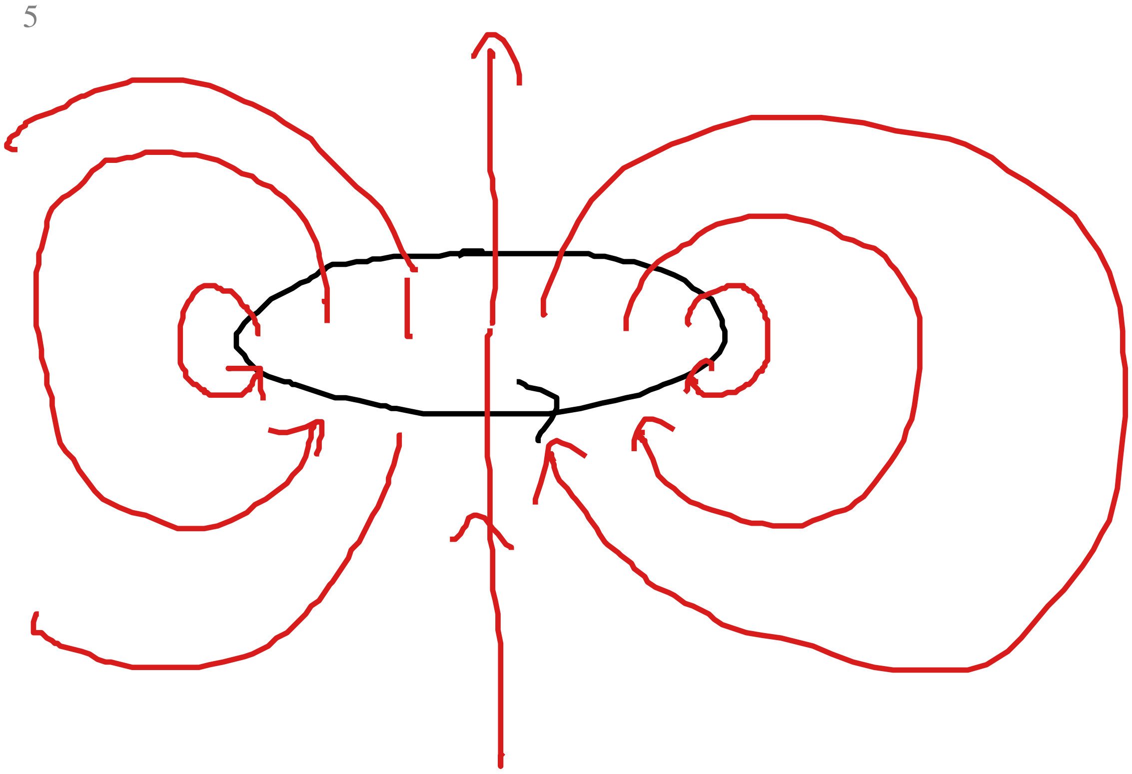
$$\vec{B} = \frac{\mu_0 I R^2}{2R^3}\hat{z} = \frac{\mu_0 I}{2R}\hat{z}$$ at center of loop

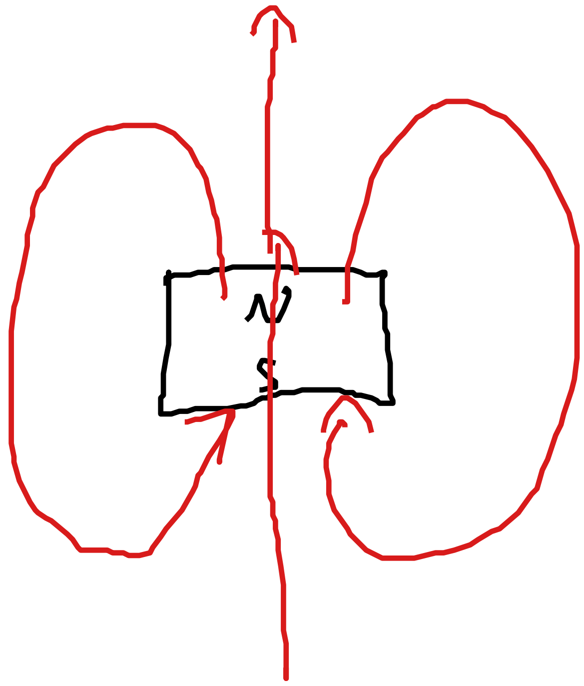
$H \gg R$

$$(R^2 + H^2)^{3/2} \simeq (H^2)^{3/2} = H^3$$

$$\vec{B} = \frac{\mu_0 \boxed{I R^2}}{2H^3}\hat{z}$$ far from loop

$$\vec{B} \sim \frac{1}{d^3}$$ dipole field





Loops of wire create magnetic dipole fields just as bar magnets do

magnetic dipole moment $\vec{\mu} = I\vec{A}$ $= I\pi R^2 \hat{z}$

for my circle

$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi H^3}$$

along axis far from dipole

any flat loop or any bar magnet

eg.

A small bar magnet creates 1T field 10cm along axis

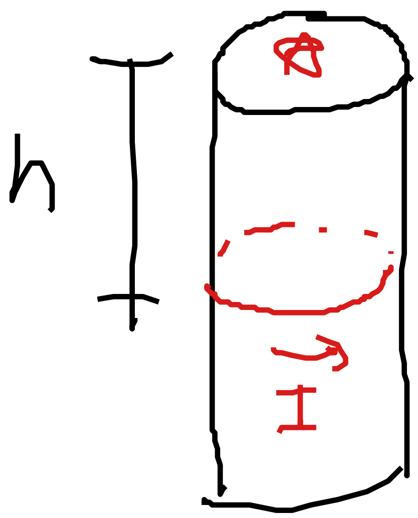
What is $\vec{\mu}$?

$$1T = \frac{\mu_0 \vec{\mu}}{2\pi (0.1m)^3}$$

$$\vec{\mu} = \frac{\cancel{2}\pi (0.001)}{{}_{2}\cancel{4}\pi \times 10^{-7}}$$

$$\vec{\mu} = \frac{10^{-3}}{2 \times 10^{-7}} = 5000\,Am^2$$

If I wrap current around a material . .



In other materials replace μ_0 (permeability of free space) with μ : permeability

paramagnet: $\mu \gtrsim \mu_0$
diamagnet: $\mu < \mu_0$

usually $\mu \simeq \mu_0$

ferromagnet: $\mu >> \mu_0$

iron: $\mu = 5000 \mu_0$

with iron core: $\vec{B} = \frac{\mu \vec{\mu}}{2\pi H^3}$

magnetic dipole moment

$H >>$ size