Physics 2140 Homework #5

5 problems

Solutions

▷ **1.**

Consider a proton (charge $e = 1.6 \times 10^{-19}$ C) and an electron (charge $-e$). Assume $PE_\infty = 0$.

(a) Before doing any calculations, would the potential energy increase or decrease if the two charges are moved farther apart?
(b) What is their potential energy if they are separated by a distance of 10^{-10} m?
(c) What is their potential energy if they are separated by twice the distance?
(d) If the electron moves from the first distance to the second, find ΔPE. Does the potential energy increase or decrease?
(e) Suppose $PE_\infty = 5 \times 10^{-18}$ J instead. How does your answer to part (d) change?

Answer:

(a) The two charges are attracted to each other, so moving them farther apart is what they would *not* do spontaneously; thus the potential energy should $\boxed{\text{increase}}$.

(b) The potential energy of two point charges is

$$PE = k\frac{q_1 q_2}{d} + PE_\infty$$

In this case $PE_\infty = 0$, and

$$PE = (9 \times 10^9)\frac{(1.6 \times 10^{-19}\,\text{C})(-1.6 \times 10^{-19}\,\text{C})}{10^{-10}} = \boxed{-2.30 \times 10^{-18}\,\text{J}}$$

(c) If the two particles are twice as far above, so that $d = 2 \times 10^{-10}$ m, then

$$PE = (9 \times 10^9)\frac{(1.6 \times 10^{-19}\,\text{C})(-1.6 \times 10^{-19}\,\text{C})}{2 \times 10^{-10}} = \boxed{-1.15 \times 10^{-18}\,\text{J}}$$

This is half the value in part (b), but only because $PE_\infty = 0$; you could choose a different baseline so that (c) would be one-third of (b), or one-tenth, or whatever you like.

(d) The change in potential energy is the final minus the initial. Thus if we move the electron from the first position to the second, then

$$\Delta PE = -1.15 \times 10^{-18}\,\text{J} - (-2.30 \times 10^{-18}\,\text{J}) = \boxed{1.15 \times 10^{-18}\,\text{J}}$$

This is positive, which means the potential energy increases, as predicted in part (a).

(e) If we change the baseline, then (b) becomes 2.70×10^{-18} J and (c) becomes 3.85×10^{-18} J, but their difference part (d) $\boxed{\text{remains the same}}$: $\Delta PE = 1.15 \times 10^{-18}$ J. Differences in potential energy are the physically relevant quantity, and the choice of baseline, being an arbitrary choice, can't change the physics of the situation.

▷ **2.**
When an electron moves from some location A to some other location B, the electric field does 3.94×10^{-19} J of work on it. Find the potential difference $\Delta V = V_B - V_A$ between the two points.

Answer:
If the field does work on the charge, then the charge must be losing potential energy, specifically $\Delta PE = -3.94 \times 10^{-19}$ J. The potential difference between the two points which the electron occupies is given by

$$\Delta V = \frac{\Delta PE}{q} = \frac{-3.94 \times 10^{-19}\,\mathrm{J}}{-1.6 \times 10^{-19}\,\mathrm{C}} = 2.46\,\mathrm{V}$$

Thus the electron is moving to a *higher* potential (because the change in potential is positive).

▷ **3.**
An *electron-volt* ($1\,\mathrm{eV}$) is the energy an electron (charge -1.6×10^{-19} C) gains by moving through a potential difference of 1 volt.

(a) How many joules is this?
(b) How many electron volts of energy does it take to pull an electron away from the proton in a hydrogen atom? Assume the electron starts off 1.05×10^{-10} m from the proton, and we end up with the electron very far away from the proton (basically at infinity).

Answer:

(a) When a charge q moves through a potential difference of ΔV it gains potential energy $\Delta PE = q\Delta V$. If an electron is to *gain* potential energy, it must move to *lower* potential (rolling downhill is what it *doesn't* like to do), so $\Delta V = -1\,\mathrm{V}$ in this case, in which case the electron gains energy

$$\Delta PE = (-1.6 \times 10^{-19}\,\mathrm{C})(-1\,\mathrm{V}) = 1.6 \times 10^{-19}\,\mathrm{J}$$

And therefore $\boxed{1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}}$.

(b) We can think of the proton as the *source* and the electron (the moving charge) as the *target*.

First, the proton creates a potential of a point charge. Assuming that $V_\infty = 0$, the potential created by the proton is $V = k\frac{e}{d}$, where e is the charge of the proton and d is the distance from the proton. If the electron starts out $1.05 \times 10^{-10}\,\mathrm{m}$ from the proton, then it is starting at potential

$$V = (9 \times 10^9\,\mathrm{Nm^2/C^2})\frac{1.6 \times 10^{-19}\,\mathrm{C}}{1.05 \times 10^{-10}\,\mathrm{m}} = 13.7\,\mathrm{V}$$

When it moves to infinity, it is at zero potential (because we said $V_\infty = 0$). Therefore to tear the electron away from the proton requires you to move it through a potential difference of $\Delta V = 0\,\mathrm{V} - 13.7\,\mathrm{V} = -13.7\,\mathrm{V}$, and since it takes $1\,\mathrm{eV}$ of energy to move an electron through one volt of potential difference, it requires $\boxed{13.7\,\mathrm{eV}}$ of energy to tear the electron away.

▷ **4.**

What is the potential $2\,\mathrm{m}$ from a $4\,\mu\mathrm{C}$ charge if

(a) the potential at infinity is zero?

(b) the potential at infinity is $5\,\mathrm{V}$?

(c) the potential 1 meter from the charge is $5\,\mathrm{V}$?

Answer:

The potential a distance d from a point charge is

$$V = k\frac{q}{d} + V_\infty$$

(a)

$$V = (9 \times 10^9)\frac{4 \times 10^{-6}}{2} + 0 = \boxed{18\,\mathrm{kV}}$$

(b)

$$V = (9 \times 10^9)\frac{4 \times 10^{-6}}{2} + 5 = \boxed{18.005\,\mathrm{kV}}$$

(c) In this case we are not told what V_∞ is, but have to solve for it. We know that the potential is $5\,\mathrm{V}$ if $d = 1\,\mathrm{m}$, so

$$5 = k\frac{q}{d} + V_\infty \implies V_\infty = 5 - k\frac{q}{d} = 5 - (9 \times 10^9)\frac{4 \times 10^{-6}}{1} = -35.995\,\mathrm{kV}$$

and therefore

$$V(d = 2\,\mathrm{m}) = (\times 10^9)\frac{4 \times 10^{-6}}{2} - 35.995\,\mathrm{kV} = 18\,\mathrm{kV} - 35.995\,\mathrm{kV} = -17.995\,\mathrm{kV}$$

You were expecting $2.5\,\mathrm{V}$ maybe? :) Potential is only inversely proportional to d when $V_\infty = 0$, not in general.

▷ **5.**
Four negative charges, $q = -3\,\mathrm{nC}$, sit on the four corners of a square with side $a = 1\,\mathrm{m}$. Find the potential at the center of the square, if $V_\infty = 0\,\mathrm{V}$. Also, what is the electric field at the center of the square?

Answer:
If $V_\infty = 0$, then the potential at a point due to several point charges is the sum of the potentials due to each individually. In each case, the four charges are all $1/\sqrt{2} = 0.71\,\mathrm{m}$ from the target, so the potential at the center due to one is

$$V_1 = k\frac{q}{d} = (9 \times 10^9)\frac{(-3 \times 10^{-9})}{0.71} = -38.03\,\mathrm{V}$$

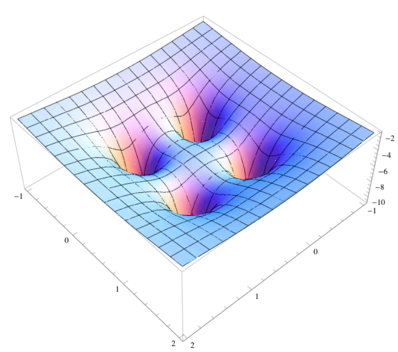
and so the total potential at the center is four times this, or $\boxed{-152\,\mathrm{V}}$.

The electric field at the center of the square is zero: the field of each charge is cancelled out by the electric field from the charge on the opposite corner. Remember that the electric field is the negative derivative (slope) of the potential: the center must be a flat spot point of minimum or maximum potential.

For your amusement, here's a plot of the potential for this configuration in the xy–plane, which I got by plotting

$$V(x,y) = -kq\left(\frac{1}{\sqrt{x^2+y^2}} + \frac{1}{\sqrt{(x-1)^2+y^2}} + \frac{1}{\sqrt{x^2+(y-1)^2}} + \frac{1}{\sqrt{(x-1)^2+(y-1)^2}}\right)$$

using *Mathematica*: Our interpretation of negative charges as "holes" seems a little more apt



after seeing this graph, no?