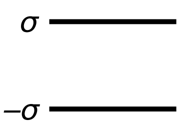
Physics 2140 Homework #6

4 problems

Solutions

▷ **1.**



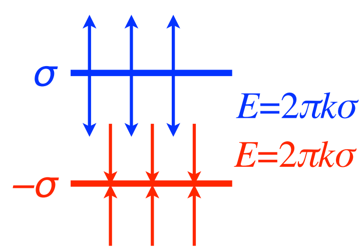
The electric field of an infinite plane with surface charge density σ, as we saw earlier, is $2\pi k\sigma$.

(a) Suppose we have two parallel infinite planes, the top one with surface charge density σ and the bottom with surface charge density $-\sigma$. What is the electric field (magnitude and direction) between the two planes?

(b) Find the potential difference between the two plates, as a function of σ, if the plates are a distance d apart.

(c) If the space between the plates is filled with air, what value of σ would cause a spark to fly across the gap between the plates? (See your notes on lightning.)

Answer:____



(a) The electric field created by each infinite plane is $2\pi k\sigma$, a constant. The electric field points away from the positive plate (so downward in between the two), and towards the negative plate (also downward), so the net electric field between the plates is $\boxed{4\pi k\sigma}$.

(b) Call the bottom plate $y = 0$ and the top plate $y = d$, so that up is the positive $+y$ direction. The electric field is constant, and $E_y = -4\pi k\sigma$. The potential difference is

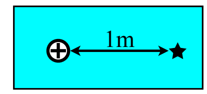
$$\begin{aligned}
\Delta V = V(d) - V(0) &= -\int_{y=0} y = dE_y\,dy \\
&= -\int_0^d (-4\pi k\sigma)\,dy \\
&= 4\pi k\sigma \int_0^d dy \\
&= \boxed{4\pi k\sigma d}
\end{aligned}$$

(c) A spark will fly through air if the electric field exceeds $E_b = 3 \times 10^6\,\mathrm{N/C}$. The electric field occurs if

$$E_b = 4\pi k\sigma \implies \sigma = \frac{E_b}{4\pi k} = \frac{3 \times 10^6\,\mathrm{N/C}}{4\pi(9\times 10^9\,\mathrm{Nm^2/C^2})} = \boxed{27\,\mu\mathrm{C/m^2}}$$

▷ **2.**

A positive charge $2\,\mathrm{mC}$ sits in water ($\kappa = 80$). What is the magnitude of the electric field $1\,\mathrm{m}$ from the charge?



Answer:

If the water weren't there, the electric field at the star would be

$$E = k\frac{q}{d^2} = (9\times 10^9\,\mathrm{Nm^2/C^2})\frac{2\times 10^{-3}\,\mathrm{C}}{(1\,\mathrm{m})^2} = 18\,\mathrm{MN/C}$$

The water reduces the electric field by a factor of $\kappa = 80$, to a magnitude of $\boxed{225\,\mathrm{kN/C}}$.

▷ **3.**

What is the net charge on a conducting sphere of radius $r = 0.15\,\mathrm{m}$ if the potential of the sphere is $1500\,\mathrm{V}$ and $V = 0$ at infinity?

Answer:

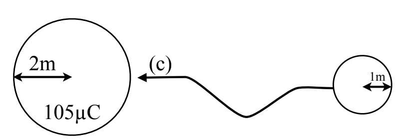
The net charge on this conductor will cover only its outer surface, forming a spherical shell with uniform charge distribution. Now the electric field outside a spherical shell of charge Q is identical to the electric field due to a point charge Q at the center of the sphere, and the same holds true for a potential. Thus the potential of this spherical shell is $V = k\frac{Q}{r}$, when $r \geq 0.15\,\mathrm{m}$ and $V = 0$ at infinity. But we know the potential right at the surface of the conductor is $1500\,\mathrm{V}$; therefore

$$1500\,\mathrm{V} = k\frac{Q}{0.15\,\mathrm{m}} \implies Q = \frac{1}{k}(1500\,\mathrm{V})(0.15\,\mathrm{m}) = 2.50\times 10^{-8}\,\mathrm{C} = \boxed{25.0\,\mathrm{nC}}$$

Note that the potential at the center of this conductor is equal to the potential at the surface of the conductor (i.e. $1500\,\mathrm{V}$) because a conductor is a equipotential volume.

▷ **4.**

A sphere with radius $2\,\mathrm{m}$ has a total charge of $105\,\mu\mathrm{C}$ on it.



(a) What is the sphere's capacitance?
(b) What is the potential of the sphere, if $V_\infty = 0\,\mathrm{V}$?

(c) Now suppose a 1 m sphere is connected to the first sphere with a long wire as shown. The smaller sphere is initially neutral. Once the wire is connected and the system reaches equilibrium, how much charge ends up on the smaller sphere? (Or alternatively, what is the ratio of the final charge on the small sphere to the final charge on the big sphere?)

Answer:____

(a) A sphere's capacitance is

$$C = \frac{R}{k} = \frac{2\,\mathrm{m}}{9 \times 10^9\,\mathrm{m/F}} = \boxed{2.2 \times 10^{-10}\,\mathrm{F}} \text{ (or } 0.22\,\mathrm{nF})$$

(b) The relationship between the potential of the sphere and the charge on it is

$$Q = C(V - V_\infty) \implies V = \frac{Q}{C} = \frac{105 \times 10^{-6}\,\mathrm{C}}{2.2 \times 10^{-10}\,\mathrm{F}} = \boxed{477\,\mathrm{kV}}$$

(c) When the spheres are first connected, the potential of the large sphere is higher than the small sphere's potential (because it is filled with positive charge), which causes positive charge to flow into the smaller sphere. The charge will stop flowing when the potentials of the two spheres are equal. If we let V_L be the potential of the large sphere and V_S be the potential of the small one, then $V_L = V_S$ at equilibrium, and $V_L - V_\infty = V_S - V_\infty$ as well. However, we know that $V - V_\infty = Q/C$ for both spheres, so

$$\begin{aligned} V_L - V_\infty &= V_S - V_\infty \\ \implies \frac{Q_L}{C_L} &= \frac{Q_S}{C_S} \\ \implies \frac{Q_L}{Q_S} &= \frac{C_L}{C_S} \end{aligned}$$

That is, the ratio of the spheres' final charges is equal to the ratio of their capacitances. (This is true for any two metal shapes, not just spheres.) For a sphere, the capacitance is proportional to its radius: thus the large sphere has twice the capacitance of the small one, and so it ends up with twice the charge as well: $Q_L = 2Q_S$. If the total charge is $Q_L + Q_S = 105\,\mu\mathrm{C}$, so

$$105\,\mu\mathrm{C} = Q_L + Q_S = 2Q_S + Q_S = 3Q_S \implies Q_S = \frac{105\,\mu\mathrm{C}}{3} = \boxed{35\,\mu\mathrm{C}}$$