Physics 2140 Homework #7

3 problems

Solutions

▷ **1.**

A metal cone has a capacitance of $C = 10\,\mu\text{F}$. Suppose one end of a power supply is connected to the cone and the other to a point far away from the cone ("ground"). What voltage should the power supply be set to, so that the cone ends up with a charge of $Q = 5\,\mu\text{C}$?

Answer:

The charge on the cone will be $Q = C\Delta V$ where ΔV is the potential difference between infinity and the cone. Thus we need

$$\Delta V = \frac{Q}{C} = \frac{5\,\mu\text{C}}{10\,\mu\text{F}} = \boxed{0.5\,\text{V}}$$

▷ **2.**

Two identical circular metal plates are separated by a distance of $10^{-9}\,\text{m}$. If these plates have a capacitance of $1\,\text{F}$, what is their radius?

Answer:

The capacitance of two parallel plates is

$$C = \epsilon_0 \frac{A}{d}$$

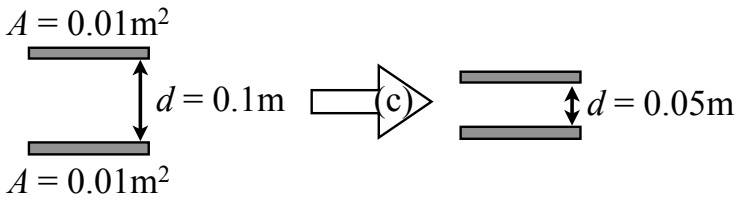
where $A = \pi R^2$ is the area of a circular disk. Solve for R:

$$\begin{aligned} C &= \epsilon_0 \frac{\pi R^2}{d} \\ \implies R^2 &= \frac{Cd}{\pi\epsilon_0} \\ \implies R &= \sqrt{\frac{Cd}{\pi\epsilon_0}} \\ &= \sqrt{\frac{(1\,\text{F})(10^{-9}\,\text{m})}{\pi(8.85\times10^{-12}\,\text{F/m})}} \\ &= \boxed{6.0\,\text{m}} \end{aligned}$$

This is about the size of a small classroom: not astronomical, but rather too large to be practical. (Using a dielectric would help.)

▷ **3.**

The figure shows two plates, each with area $A = 0.01\,\mathrm{m}^2$, which are $d = 0.1\,\mathrm{m}$ apart. They are connected by a $9\,\mathrm{V}$ battery.



(a) Find the capacitance of this capacitor.
(b) Find the energy stored in the capacitor.
(c) If the two plates are moved closer together, so that they are only $d = 0.05\,\mathrm{m}$ apart, what is the energy stored in the capacitor then?

Answer:

(a) The capacitance of a parallel-plate capacitor is

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\,\mathrm{F/m})\frac{0.01\,\mathrm{m}^2}{0.1\,\mathrm{m}} = \boxed{8.85 \times 10^{-13}\,\mathrm{F}}$$

(b) The energy stored in the capacitor is

$$PE = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(8.85 \times 10^{-13}\,\mathrm{F})(9\,\mathrm{V})^2 = \boxed{3.58 \times 10^{-11}\,\mathrm{J}}$$

(c) Halving the distance d between the plates will double the capacitance, because $C = \epsilon_0 \frac{A}{d}$. If the plates are moved closer together **while connected to the battery**, then $\Delta V = 9\,\mathrm{V}$ still, and so

$$PE = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(2 \times 8.85 \times 10^{-13}\,\mathrm{F})(9\,\mathrm{V})^2 = \boxed{7.17 \times 10^{-11}\,\mathrm{J}}$$

However, if the plates were disconnected from the battery before being moved closer together, then their charge remains the same. In that case, the equation $PE = \frac{Q^2}{2C}$ tells us that doubling the capacitance should halve the potential energy stored in the capacitor, and so $PE = 1.79 \times 10^{-11}\,\mathrm{J}$.

Be sure you're clear on the distinction between these two cases.