Physics 2140 Homework #8

4 problems

Solutions

▷ **1.**

A cylindrical wire of length $1\,\mathrm{m}$ and radius $5\,\mathrm{mm}$ is made of copper, which has a resistivity of $\rho = 17 \times 10^{-9}\,\Omega \cdot \mathrm{m}$. One end of the wire is $5\,\mathrm{V}$ higher than the other.
(a) Find the resistance of the wire.
(b) Find the current through the wire.
(c) Find the power dissipated by the wire.

Answer:

(a) The resistance of a wire is

$$R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} = (17 \times 10^{-9}\,\Omega \cdot \mathrm{m}) \frac{1\,\mathrm{m}}{\pi (0.005\,\mathrm{m})^2} = 2.16 \times 10^{-4}\,\Omega = \boxed{0.21\,\mathrm{m\Omega}}$$

(b) The current through the wire is given by Ohm's Law

$$I = \frac{\Delta V}{R} = \frac{5\,\mathrm{V}}{2.16 \times 10^{-4}\,\Omega} = \boxed{23\,\mathrm{kA}}$$

which is huge.

(c)

$$P = I \Delta V = (23\,\mathrm{kA})(5\,\mathrm{V}) = 1.15 \times 10^5\,\mathrm{W} = \boxed{115\,\mathrm{kW}}$$

▷ **2.**

A human being can be electrocuted if a current as small as $50\,\mathrm{mA}$ passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding, one in each hand. If his resistance is $2000\,\Omega$, what might the fatal voltage be?

Answer:

For a resistance $R = 2000\,\Omega$, the potential difference required to create a current $I = 50\,\mathrm{mA}$ is

$$\Delta V = IR = (50\,\mathrm{mA})(2000\,\Omega) = \boxed{100\,\mathrm{V}}.$$

So a $100\,\mathrm{V}$ potential difference (less than the $110\,\mathrm{V}$ in normal electrical outlets) could be enough to be fatal. (Note that the resistance of skin can vary significantly depending on if it is wet or dry: from $100\,\Omega$ for skin soaked in salt water, to $500\,\mathrm{k\Omega}$ for very dry skin.)

▷ **3.**

A 40-watt incandescent light bulb is so called because when connected to a 120 V power supply, it emits 40 watts of power, not all of it as light.

(a) Find the resistance of a 40 W bulb and a 60 W bulb.

(b) Find the current through each, if they are connected separately to a 120 V power supply.

(c) How would the power output of a 40 W light bulb change if it were used in a country where 220 V is the norm? Would it be brighter or dimmer?

Answer:

(a) The power output of a resistor is $P = I\Delta V$; because $I = \frac{\Delta V}{R}$ according to Ohm's Law, we can either write the power as

$$P = I^2 R \quad \text{or} \quad P = \frac{(\Delta V)^2}{R}$$

We choose one equation or the other, depending on whether we know I or ΔV. In this case, we hook both light bulbs up to the same $120\,\mathrm{V}$ power supply, so ΔV is the same for both, so we use the form

$$P = \frac{(\Delta V)^2}{R} \implies R = \frac{(\Delta V)^2}{P}$$

and the resistances are

$$R_{40} = \frac{(120)^2}{40} = 360\,\Omega \quad \text{and} \quad R_{60} = \frac{(120)^2}{60} = 240\,\Omega$$

Note that the higher-wattage bulb has the lower resistance.

(b) Since the power output is $P = I\Delta V$ and the drop in potential across each is $120\,\mathrm{V}$, the current through each is $I = \frac{P}{\Delta V}$, or

$$I_{40} = \frac{40\,\mathrm{W}}{120\,\mathrm{V}} = 0.33\,\mathrm{A} \quad \text{and} \quad I_{60} = \frac{60\,\mathrm{W}}{120\,\mathrm{V}} = 0.5\,\mathrm{A}$$

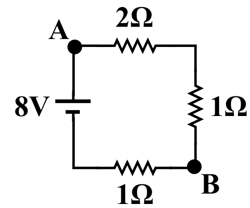
(c) The resistance of the bulb is what stays constant as we go from one country to another: the $40\,\mathrm{W}$ light bulb has a resistance of $360\,\Omega$. The power output is $P = (\Delta V)^2/R$, so when $\Delta V = 220\,\mathrm{V}$,

$$P = \frac{(220\,\mathrm{V})^2}{360\,\Omega} = 134\,\mathrm{W}$$

So the light bulb will be much brighter (unless it burns out). Notice that the power is proportional to the potential difference squared, so a doubling of the voltage quadruples the power output.

▷ **4.**

The figure shows an 8 V battery connected in series with a $2\,\Omega$ resistor and two $1\,\Omega$ resistors.



(a) Find the current through the circuit.
(b) Find the drop in potential ΔV across the 2Ω resistor.
(c) Find the potential difference between the points marked A and B.

Answer:

(a) We use the loop rule here, assuming the current through the circuit is I clockwise. Start at the negative terminal of the battery and move clockwise: we go up the battery $(+8)$, down the first resistor $(-2I)$, down the second $(-1I)$, and down the third $(-1I)$, before returning to the starting point. Thus

$$0 = 8 - 2I - 1I - 1I \implies 8 = 4I \implies \boxed{I = 2\,\mathrm{A}}$$

(b) The potential drop across a resistor is given by Ohm's Law: $\Delta V = IR = (2\,\mathrm{A})(2\,\Omega) = \boxed{4\,\mathrm{V}}$.

(c) If we start at A and go clockwise, then we go down the first resistor $(-2I)$ and down the second $(-1I)$ and get to B. Thus the potential difference between A and B is

$$\Delta V = V_B - V_A = -2I - 1I = -(2\,\Omega)(2\,\mathrm{A}) - (1\,\Omega)(2\,\mathrm{A}) = -6\,\mathrm{V}$$

Thus the potential difference between the two points is $\boxed{6\,\mathrm{V}}$; more specifically $V_B = V_A - 6\,\mathrm{V}$, because V_B is lower than V_A.