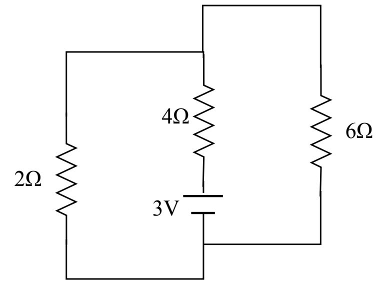
Physics 2140 Homework #9

4 problems

Solutions

▷ **1.**

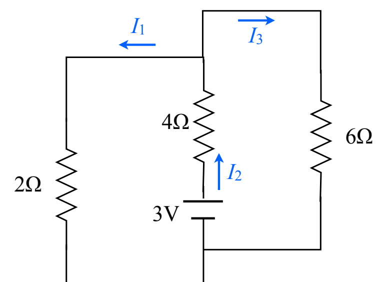
Find the current through all three resistors.



Answer:

First thing to do is to label all three currents, as shown. (You could have numbered them differently, of course.) We have three unknowns, so we need three equations to solve for them, a combination of loop rules and junction rules. There is one junction rule:

$$\boxed{I_2 = I_1 + I_3 \text{ [A]}}$$



which is determined by looking at the junction right above the $4\,\Omega$ resistor. There are three loop rules we could write. The easiest one to work with (because it has the fewest components) is the loop that passes through the $2\,\Omega$ and $6\,\Omega$ resistors:

$$-2I_1 + 6I_3 = 0 \implies \boxed{I_1 = 3I_3 \text{ [B]}}$$

The other two loop rules are

$$3 - 4I_2 - 2I_1 = 0 \quad \text{and} \quad \boxed{3 - 4I_2 - 6I_3 = 0 \text{ [C]}}$$

I'll use the second of these as my third equation.

To solve three equations in three unknowns, we pick one of the three variables, and write the other two variables in terms of this one. In this case I'm going to pick I_3, because from equation [B] I see that $I_1 = 3I_3$. Substituting this result into equation [A] gives me that

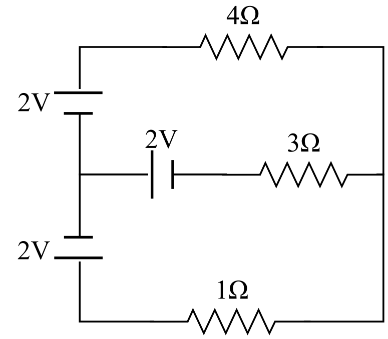
$$I_2 = 3I_3 + I_3 \implies I_2 = 4I_3$$

And substituting this result into equation [C] gives me

$$3 - 4(4I_3) - 6I_3 = 0 \implies 3 = 16I_3 + 6I_3 \implies \boxed{I_3 = \frac{3}{22} = 0.136\,\text{A}}$$

Now that I know I_3, I can go back and find the other two currents: $\boxed{I_2 = 4I_3 = 0.545\,\text{A}}$ and $\boxed{I_1 = 3I_3 = 0.409\,\text{A}}$

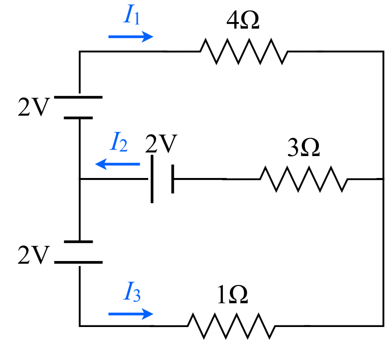
▷ **2.**
Find the current through all three batteries. Note that the current will run backward through at least one battery; assume that this can happen without any difficulty (it will end up recharging the battery).



Answer:

Again, we label the currents, and find three equations for three unknowns. The junction rule is

$$\boxed{I_2 = I_1 + I_3 \text{ [A]}}$$



The top loop gives us

$$2 + 2 - 4I_1 - 3I_2 = 0 \implies \boxed{4 = 4I_1 + 3I_2 \text{ [B]}}$$

and the bottom loop gives us

$$2 + 2 - 1I_3 - 3I_2 = 0 \implies \boxed{4 = I_3 + 3I_2 \text{ [C]}}$$

Let's write I_1 and I_3 in terms of I_2. Equation [B] shows us that

$$I_1 = 1 - \frac{3}{4}I_2$$

and equation [C] shows us that

$$I_3 = 4 - 3I_2$$

If we substitute these into equation [A] we get

$$I_2 = (1 - \frac{3}{4}I_2) + (4 - 3I_2) \implies I_2 + 0.75I_2 + 3I_2 = 5 \implies \boxed{I_2 = 1.053\,\mathrm{A}}$$

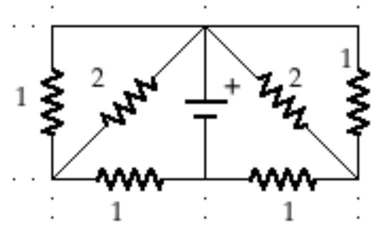
which means that

$$\boxed{I_3 = 4 - 3(1.053) = 0.841\,\mathrm{A}}$$

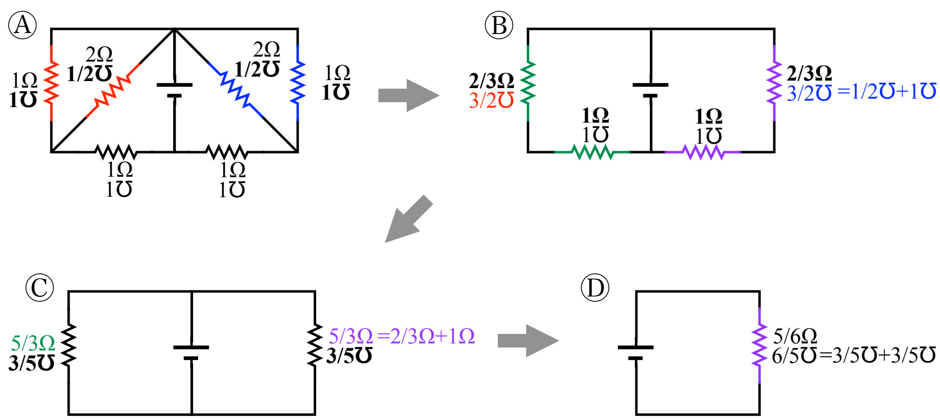
and

$$\boxed{I_1 = 1 - 0.75(1.053) = 0.210\,\mathrm{A}}$$

▷ **3.**
In the figure below, the battery has emf $\mathcal{V} = 5\,\mathrm{V}$ and the resistors are all $1\,\Omega$ or $2\,\Omega$. Find the effective resistance of all the resistors combined, and find the current through the battery.



Answer:
This can be solved by replacing combinations of resistors with their equivalents, one step at a time. The following diagram shows the basic steps.



For the first reduction (1), we note that resistors a and b are in parallel, as are e and f. Thus we replace resistors a and b with resistor g, which has resistance

$$\frac{1}{R_g} = \frac{1}{R_a} + \frac{1}{R_b} = \frac{1}{1\,\Omega} + \frac{1}{2\,\Omega} = \frac{3}{2\,\Omega}$$

or $R_g = \frac{2}{3}\,\Omega$, and similarly replace e and f with h.

In the second reduction, g and c are in series, so they are replaced with a resistor i where $R_i = R_g + R_c = \frac{2}{3} + 1 = \frac{5}{3}\,\Omega$, and similarly with h and d combined to form j.

Finally, resistors i and j are in parallel, so they can be replaced with a single resistor k where

$$\frac{1}{R_k} = \frac{1}{R_i} + \frac{1}{R_j} = \frac{3}{5} + \frac{3}{5} = \frac{6}{5\,\Omega},$$

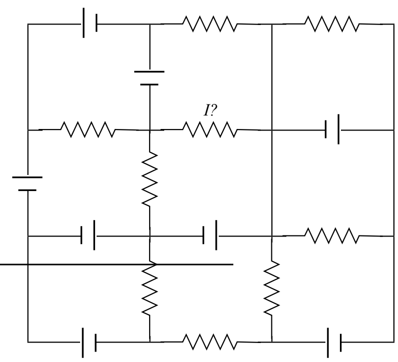
or $\boxed{R_k = \frac{5}{6}\,\Omega}$.

Finally, the current through the battery is

$$I = \frac{\mathcal{V}}{R_k} = \frac{5\,\mathrm{V}}{(5/6)\,\Omega} = \frac{30}{5}\,\mathrm{A} = \boxed{6\,\mathrm{A}}.$$

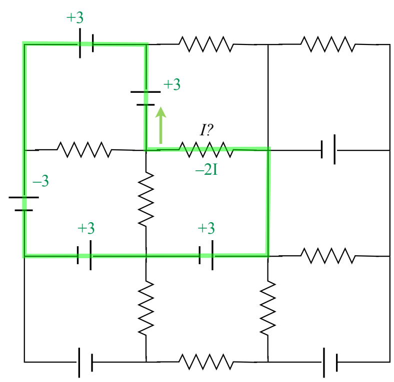
▷ **4.**

Find the current through the resistor labelled "I?", if all batteries are $3\,\mathrm{V}$ and all resistors are $2\,\Omega$. Hint: Simple loops! (Watch the Youtube video linked to on the website if you don't know what that means.)



Answer:______________________________

Solving this entire circuit would be a nightmare, so we'll rely on the hint. A *simple loop* is a loop in the circuit which only contains one resistor: its loop rule contains only one term of the form $I_n R$, which means it only has one unknown current to solve for. If we can find a simple loop through the resistor in question, we can solve for the current I directly.



The second figure shows such a loop highlighted in green. Its loop rule is

$$3 + 3 - 3 + 3 + 3 - 2I = 0 \implies 2I = 9 \implies \boxed{I = 4.5\,\mathrm{A}}$$