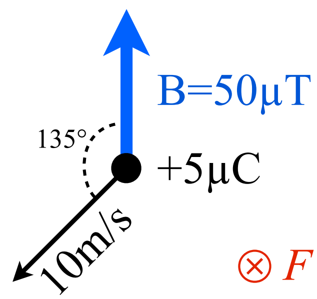
▷ **2.**

A $+5\,\mu$C charge moves southwest at $10\,$m/s in a magnetic field of $\vec{B} = 50\,\mu$T which points due north. What magnetic force does the charge feel?

Answer:____

The force on the charge is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

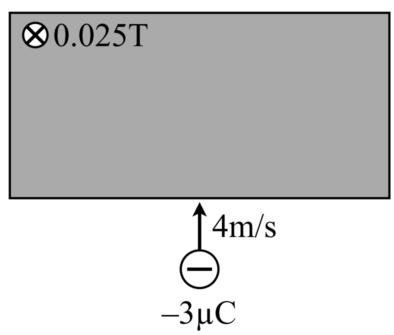


The direction must be perpendicular to both $\vec{v}$ and $\vec{B}$, and so the force points either up or down (that is, either out of or into the page: we're looking at this scenario from above). The cross-product right-hand rule indicates that the force is down. (The charge is positive so there's no "electron twist" to do.) The angle between southwest and north is 135°, and so the magnitude of the force is

$$\begin{aligned} F &= q\,|v|\,|B| \sin 135° \\ &= (5 \times 10^{-6}\,\mathrm{C})(10\,\mathrm{m/s})(50 \times 10^{-6}\,\mathrm{T}) \sin 135° \\ &= 1.77 \times 10^{-9}\,\mathrm{N} \end{aligned}$$

and so the force on the charge is $\boxed{1.77\,\mathrm{nN\ downward}}$.

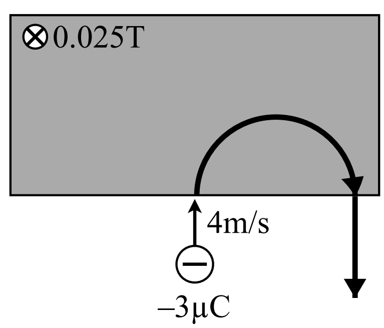
▷ **3.**
A $-3\,\mu$C charge with mass 2 mg is moving through empty space (with $B = 0$) when it hits a region with a uniform magnetic field of 0.025 T pointing into the page.
(a) Find the force at the moment the charge enters the field.
(b) Does the charge ever leave the field? If so, where? How far away? Assume the magnetic field stretches infinitely upward, to the left, and to the right.



Answer:____

(a) The force on the charge points to the right: up cross in points to the left, and then we use the "electron twist" (because the charge is negative). Since $\vec{v}$ and $\vec{B}$ are perpendicular, the magnitude of the force is simply

$$F = qvB = (3 \times 10^{-6}\,\mathrm{C})(4\,\mathrm{m/s})(0.025\,\mathrm{T}) = \boxed{3 \times 10^{-7}\,\mathrm{N}}$$

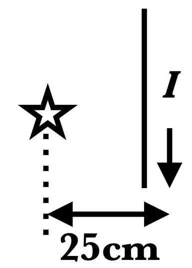


(b) The electron moves in a circular path until it returns to the $B = 0$ region, as shown on the right; once $B = 0$ it no longer feels a force and moves in a straight line. The radius of the circular path it makes is

$$r = \frac{mv}{qB} = \frac{(2 \times 10^{-6}\,\mathrm{kg})(4\,\mathrm{m/s})}{(3 \times 10^{-6}\,\mathrm{C})(0.025\,\mathrm{T})} = 107\,\mathrm{m}$$

and so the charge will leave the region $2r = 214\,\mathrm{m}$ to the right of the entry point.

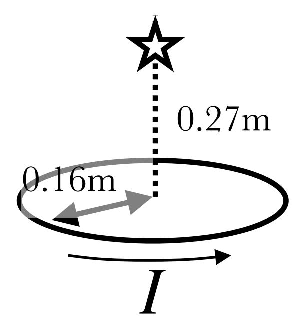
5. What is the magnitude of the magnetic field at the star, if the current in this long, straight wire is I=0.37A?



The field a distance r from a wire is given by
$\vec{B} = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\,\mu\text{T}\cdot\text{m/A})(0.37\text{A})}{2\pi(0.25\text{m})} = 0.3\mu\text{T} = 3.0\times10^{-7}\text{T}.$

Though it's more precise to write $\mu_0 = 4\pi\times10^{-7}\text{Tm/A}$, on a calculator it is more convenient to use $\mu_0 = 1.26\times10^{-6}\text{Tm/A}$ or $\mu_0 = 1.26\,\mu\text{T}\cdot\text{m/A}$.

6. This circular loop of wire has a radius of 0.16m, and carries a current of 0.45A counter-clockwise (as seen from above). What is the magnetic field (**magnitude and direction**) at the star, a distance of 0.27m above the center of the circle?

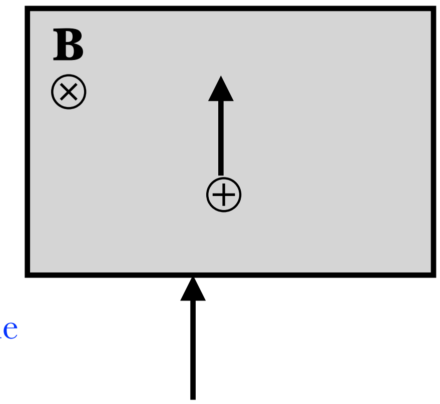


The field a distance z above the center of a loop of wire with radius R is
$\vec{B} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}} = \frac{(1.26\,\mu\text{T}\cdot\text{m/A})(0.45\text{A})(0.16\text{m})^2}{2[(0.27\text{m})^2+(0.16\text{m})^2]^{3/2}} = 0.23\mu\text{T}$ **upward.**

A lot of people missed the instruction to include the *direction* of the field. Use a circle-line RHR to find it.

If you used $\frac{\mu_0 I R^2}{2z^3}$, you were assuming that z is much larger than R, and you got the answer 0.37μT. This isn't particularly close, so apparently that assumption is false.

7. The grey area contains a magnetic field of 3.9×10^{-2}T which points into the page. A +47μC charge with a mass of 2.5×10^{-9}kg is moving at 350m/s upward. What is the force (*magnitude and direction)* on the charge due to the magnetic field?



The force on a charge in a magnetic field is equal to $F = qvB$ if the charge is moving perpendicular to the field. Thus
$F = (47\mu\text{C})(350\text{m/s})(3.9 \times 10^{-2}\text{T}) =$ $\mathbf{6.4 \times 10^{-4}\,N}$
To find the direction we use the "qvB right-hand rule": fingers point to the right, palm points into the page, thumb points up. Thus the force points **leftward** (perpendicular to both field and motion).

8. In the picture above, the charge will start spinning in a circle. What will be the radius of the circle? And will the charge spin clockwise ↻ or counterclockwise ↺?

The force will cause the charge to turn to the left, and it will keep turning to the left, taking a **counterclockwise** path. The radius of the circle is given by

$$r = \frac{mv}{qB} = \frac{(2.5 \times 10^{-9}\text{kg})(350\text{m/s})}{(47\mu\text{C})(3.9 \times 10^{-2}\text{T})} = \mathbf{0.48m} \text{ or } 48\text{cm}.$$

Physics 102 Homework #11

due Sunday, April 17th

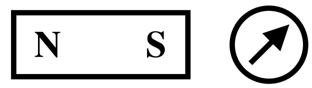
Urone Chapter 22.1–22.9

Questions (1 point each)

a) The N pole of the Earth is closer to

A) Canada **B) Australia**

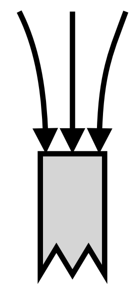
(b)



b) If a compass is placed to the right of a magnet's S pole, it will point

A) to the left ← **B) to the right →**

(d)



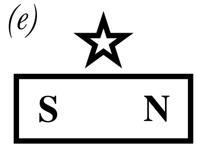
c) Which of the following materials is attracted to a magnet? Circle all that are.

A) copper **B) iron** **C) marble**

d) The picture shows the top of a bar magnet and some of its magnetic field lines. The pole shown is the

A) N pole **B) S pole**

(e)



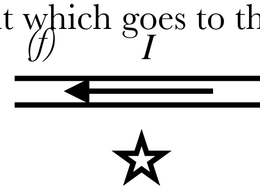
e) What is direction of the magnetic field at the star, next to this bar magnet?

A) left ← **B) right →**

C) out of the page ⊙ **D) into the page ⊗**

f) What is the direction of the magnetic field at the star, next to this current which goes to the left?

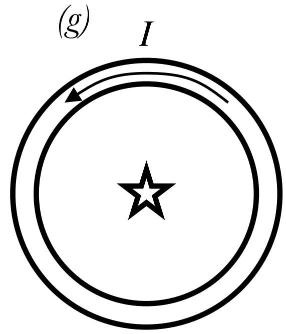
(f)



A) left ← **B) right →**

C) out of the page ⊙ **D) into the page ⊗**

(g)



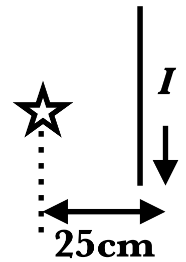
g) What is the direction of the magnetic field at the star, inside this loop of current?

A) left ← **B) right →**

C) out of the page ⊙ **D) into the page ⊗**

Problems (4 points each)

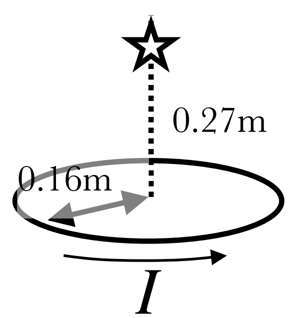
1. What is the magnitude of the magnetic field at the star, if the current in this long, straight wire is I=0.37A?



The field a distance r from a wire is given by
$\vec{B} = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\,\mu\text{T}\cdot\text{m/A})(0.37\text{A})}{2\pi(0.25\text{m})} = 0.3\mu\text{T} = 3.0\times10^{-7}\text{T}.$

Though it's more precise to write $\mu_0 = 4\pi\times10^{-7}\text{Tm/A}$, on a calculator it is more convenient to use $\mu_0 = 1.26\times10^{-6}\text{Tm/A}$ or $\mu_0 = 1.26\,\mu\text{T}\cdot\text{m/A}$.

2. This circular loop of wire has a radius of 0.16m, and carries a current of 0.45A counter-clockwise (as seen from above). What is the magnetic field (magnitude and direction) at the star, a distance of 0.27m above the center of the circle?



The field a distance z above the center of a loop of wire with radius R is
$\vec{B} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}} = \frac{(1.26\,\mu\text{T}\cdot\text{m/A})(0.45\text{A})(0.16\text{m})^2}{2[(0.27\text{m})^2+(0.16\text{m})^2]^{3/2}} = 0.23\mu\text{T}$ **upward.**

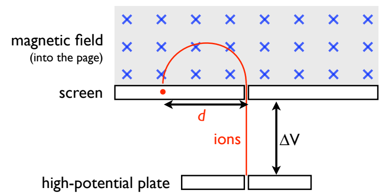
Physics 2140 Homework #11

5 problems

Solutions

▷ **1.**

A *mass spectrometer* is a device that uses electric and magnetic fields to measure the mass of atomic ions (which are atoms that have more or fewer electrons than usual, and thus have a nonzero charge). An ion is accelerated through a potential difference ΔV, after which it is allowed to enter a uniform magnetic field B, as shown in the figure. The ion then travels in a semicircular path until it hits a phosphorescent screen, a distance d from its entry point. The screen glows when an ion hits it.



Suppose we ionize a lot of neon atoms—that is, we remove one electron from them, making them positively charged—and send them through the spectrometer. Two spots show up on the screen, one 4.08 m from the entry point, and one 4.28 m from the entry point. If the potential difference the ion is accelerated through is $\Delta V = 1000\,\mathrm{V}$ and the magnetic field is 0.01 T, find the masses of the atoms that created these spots. (The spots are caused by different *isotopes* of neon; depending on the brightness of the spots, we can measure the proportions of these isotopes in our sample.)

Answer:

The distance d is equal to twice the radius of the path of the ion: $d = 2r$. The radius of the ion's path is in turn equal to $r = mv/qB$, so $d = 2mv/qB$. We know all of these quantities except for v, the speed of the ion when it enters the field. Where does it get this speed? It is moving because it has been accelerated through a potential difference ΔV (or in other words, the electric field caused the ion to accelerate). Thus, we need to calculate v as a function of ΔV. The easiest way to do that is using energy: the ion gains kinetic energy $\frac{1}{2}mv^2$, and it gets that energy by losing electrical potential energy of the amount $q\Delta V$. Thus

$$q\Delta V = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2q\Delta V}{m}}$$

and so

$$d = \frac{2m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{4m^2}{q^2B^2}}\sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{8m\Delta V}{qB^2}}$$

We can find the masses by solving the above equation for m. We know that $B = 0.01\,\mathrm{T}$ and $\Delta V = 1000\,\mathrm{V}$. The charge of the ions is equal to the charge of an electron: $q = 1.6 \times 10^{-19}\,\mathrm{C}$.

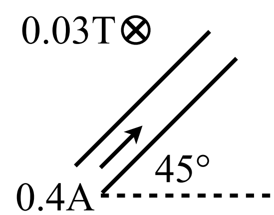
$$d^2 = \frac{8m\Delta V}{qB^2} \implies m = \frac{d^2qB^2}{8\Delta V} = d^2\frac{(1.6 \times 10^{-19}\,\mathrm{C})(0.01\,\mathrm{T})^2}{8(1000\,\mathrm{V})} = (2.0 \times 10^{-27}\,\mathrm{kg/m^2})d^2$$

The ion that lands $d = 4.08\,\mathrm{m}$ from the opening has mass $3.33 \times 10^{-26}\,\mathrm{kg}$ while the ion that lands $d = 4.28\,\mathrm{m}$ from the opening has mass $3.66 \times 10^{-26}\,\mathrm{kg}$.

It's rather informative to rewrite these masses in terms of atomic mass units (which are defined so that a Carbon–12 atom has mass $12\,\mathrm{amu}$). One amu is equal to $1.66 \times 10^{-27}\,\mathrm{kg}$, so the two ions above have mass 20 amu and 22 amu, which happen to be the two primary isotopes of neon (Neon–20 and Neon–22).

▷ **2.**

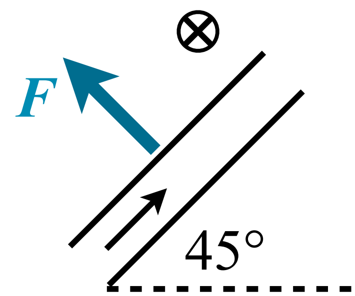
What is the force on 1 meter of a 0.4 A current that runs as shown, in a 0.03 T magnetic field that points into the page? Find the magnitude and describe the direction.



Answer:____

The force on the wire is given by the formula

$$\vec{F} = L\vec{I} \times \vec{B}$$



The magnitude of this force is

$$\left|\vec{F}\right| = L\left|\vec{I} \times \vec{B}\right| = LIB\sin\theta$$

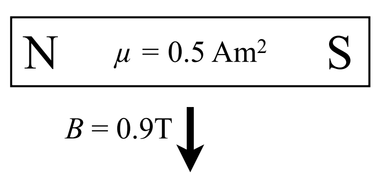
where θ is the angle between the current and the magnetic field, in this case 90°. (A vector perpendicular to a plane (like this page) is perpendicular to any vector on that plane.) Thus the magnitude of the force is

$$\left|\vec{F}\right| = LIB = (1\,\mathrm{m})(0.4\,\mathrm{A})(0.03\,\mathrm{T}) = \boxed{0.012\,\mathrm{N}}$$

The direction of the force (which can be determined with the cross-product right-hand rule) is up and to the left, as shown in the figure.

▷ **3.**

A bar magnet with dipole moment $\mu = 0.5\,\mathrm{A \cdot m^2}$ is in a magnetic field of $\vec{B} = 0.9\,\mathrm{T}$ downward, as shown. Find the torque $\vec{\tau}$ on the bar magnet.

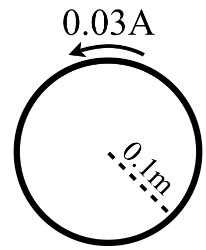


Answer:____

The torque on a magnetic dipole is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic dipole of a bar magnet points from its south pole to its north pole, to the left in this example, and so the torque on this bar magnet points $\boxed{\text{out of the page}}$ (implying a counterclockwise rotation). Because $\vec{\mu}$ is perpendicular to $\vec{B}$ here, the torque has a magnitude of $\tau = \mu B = (0.5\,\mathrm{Am}^2)(0.9\,\mathrm{T}) = \boxed{0.45\,\mathrm{Nm}}$.

▷ **4.**

Find the magnetic dipole moment $\vec{\mu}$ of a circle (radius 0.1 m) with a counterclockwise current 0.03 A.



Answer:____

The direction of the magnetic dipole moment of a current loop is found with a right-hand rule: fingers curling in the direction of the current, thumb in the direction of $\vec{\mu}$. So $\vec{\mu}$ points $\boxed{\text{out of the page}}$ here. Its magnitude is $|\vec{\mu}| = \left|I\vec{A}\right| = IA = (0.03\,\mathrm{A})(\pi(0.1\,\mathrm{m})^2) = \boxed{9.4 \times 10^{-4}\,\mathrm{A}\cdot\mathrm{m}^2}$